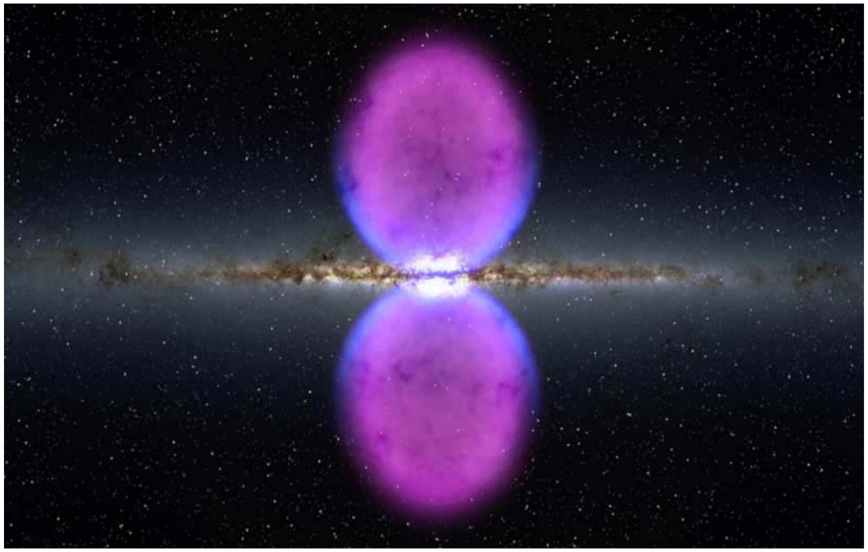


UCRIVERSIDE UNIVERSITY OF CALIFORNIA

Why do I need to know math?

Maria C. Simani

University of California, Riverside

Mathematics as a Way of Knowing

In our endeavor to understand reality we are like a man trying to understand the mechanism of a closed watch.

He sees the face and the moving hands, even hears its ticking, but he has no way of opening the case.

If he is ingenious, he may form some picture of a mechanism which could be responsible for all the things he observes,

but he may never be quite sure his picture is the only one which could explain his observations.

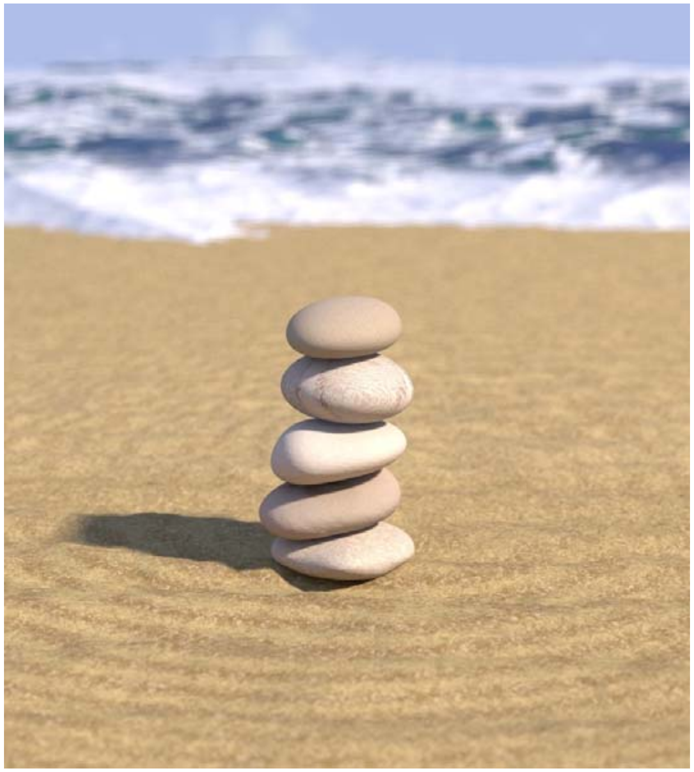
Einstein and Infeld
The Evolution of Physics

Mathematics as an Art of Knowing

At the leading edge of experience in philosophy, science and feeling there is inevitably a groping for language to translate the insecure novelty of noticing and understanding into a precision of meaning and imagery.

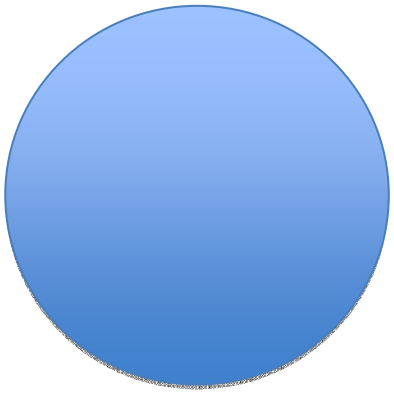
Frank Oppenheimer

- Math is the language to describe Nature
- Math allows to see the unseeable
- Math has predictive power

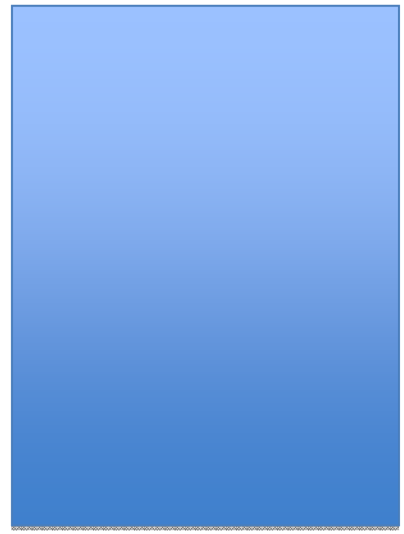


A Game of Shadows

NASA Image 1 – 8am

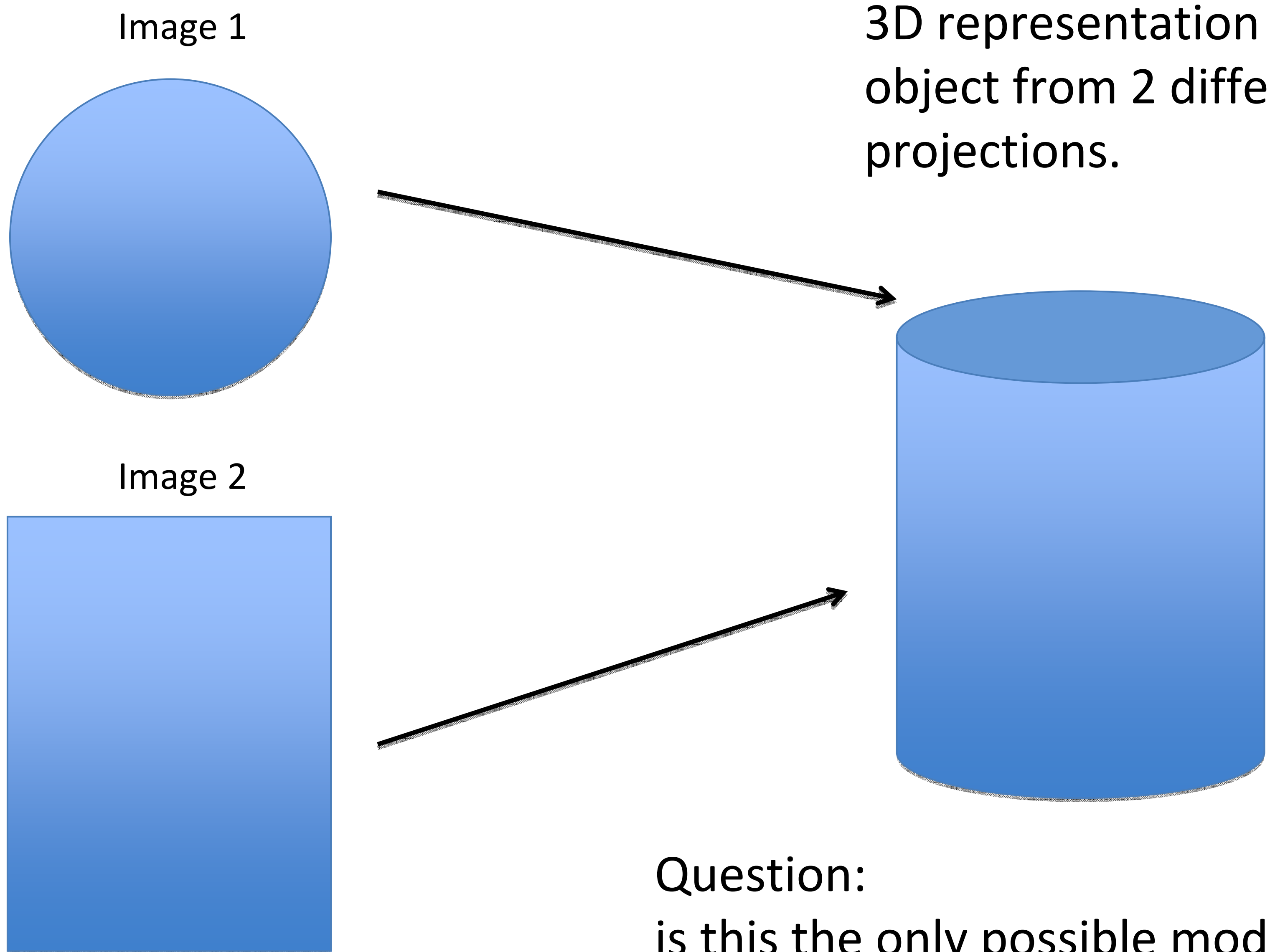


NASA Image 2 – 12pm



A Game of Shadows

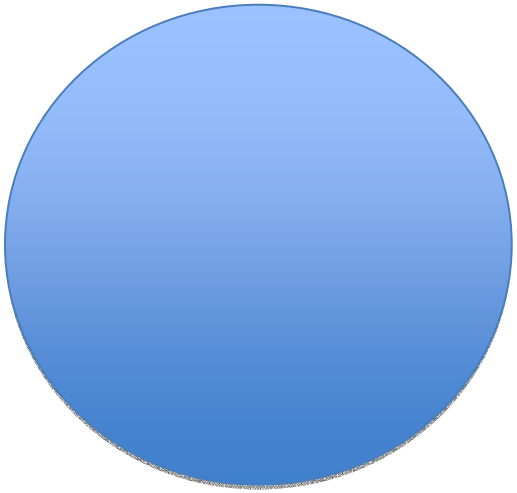
3D representation of an object from 2 different projections.



Question:
is this the only possible model?

Let’s build a cow…

An engineer, a psychologist and a physicist are called in as consultants to a dairy farm. They are tasked to make suggestions to increase milk production…



Let’s build a cow…

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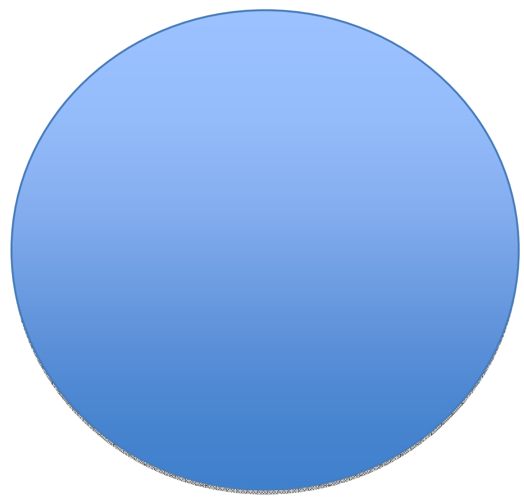
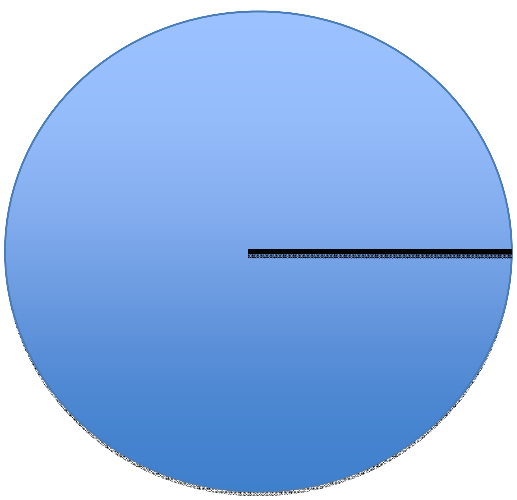


Start to think simply about the world by eliminating irrelevant details.

Let’s build a cow

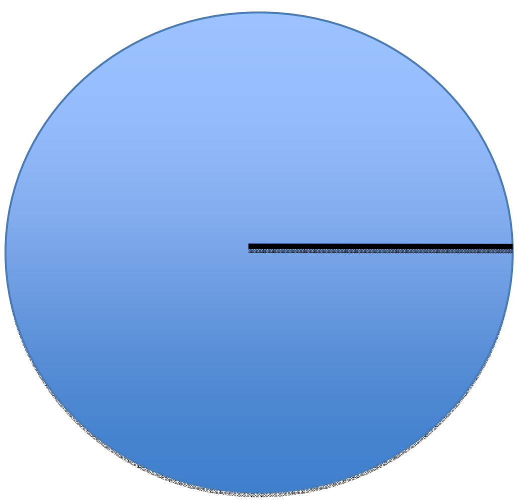
- At the core of the ability to make scientific models is the ability to reason in terms of proportions
- Scale-up, scale-down, fractions!

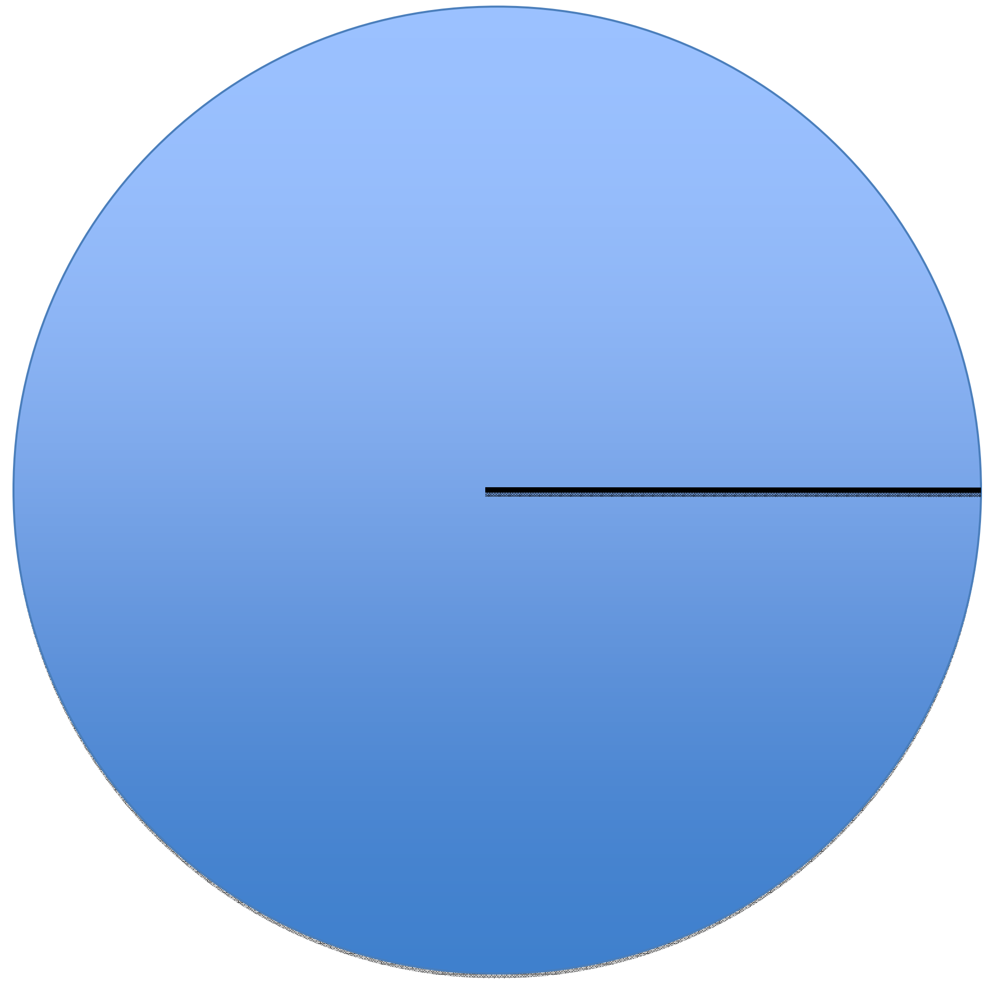


My cow is completely determined by the radius of the sphere: r

Let’s build a supercow

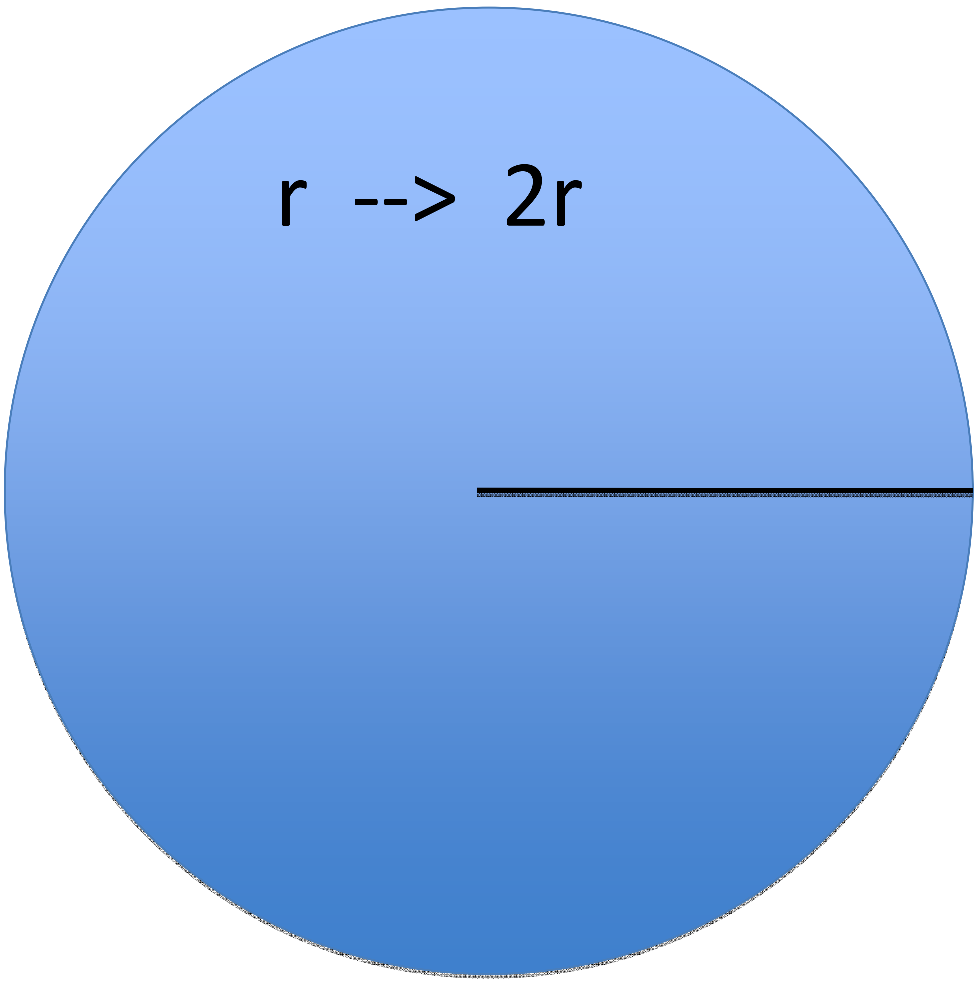
r --> 2r





What does really mean that the supercow is twice as big as a regular cow?

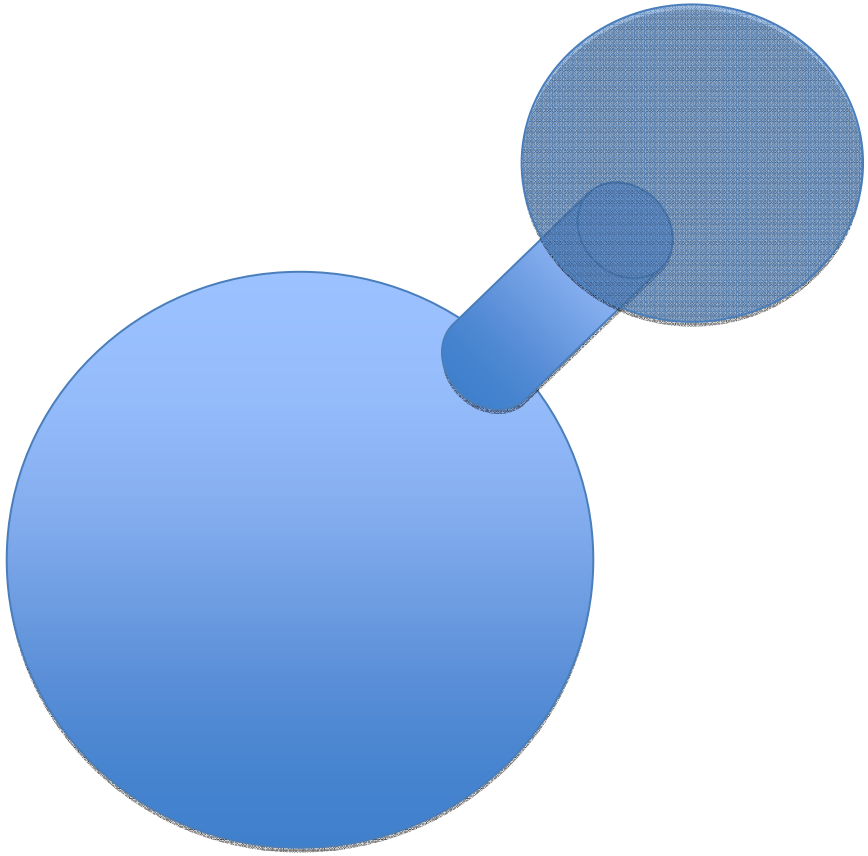
About my supercow



The supercow is:
- 8 times more heavy (volume)
- 4 times more skin (surface area)

What are the implications?

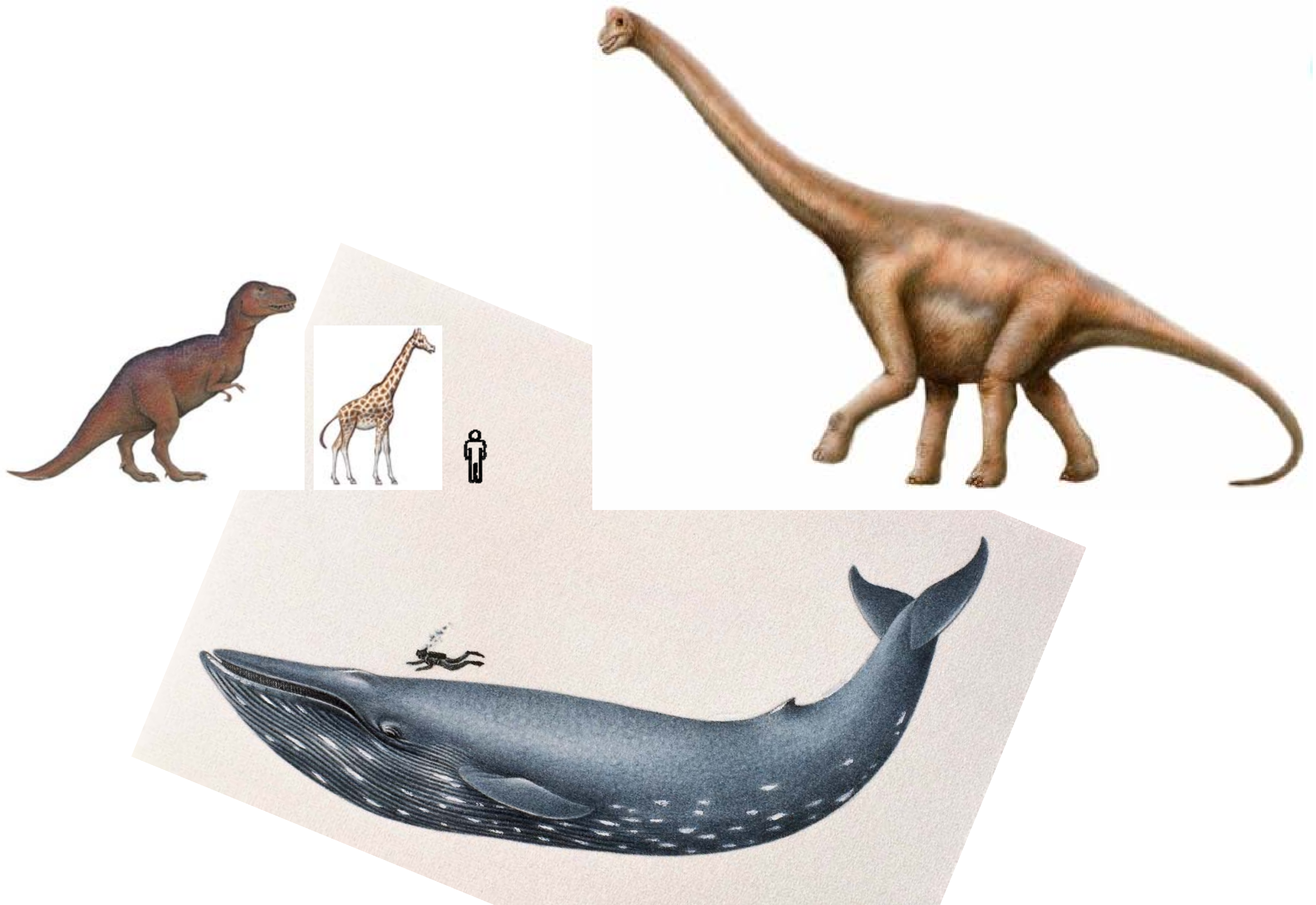
An improved cow model…



- New element in the model: the cow's neck
- The strength of the neck is proportional to its cross section
- The neck of my supercow is only 4 times stronger (surface area) to hold up a head that is 8 times heavier!
- The neck is only 50% as effective in supporting the head.

Lesson Learned from Building a Cow

- *If the only tool you have is a hammer,*
  you tend to treat everything as if it were a nail.



NASA Shuttle Launch (modeling)

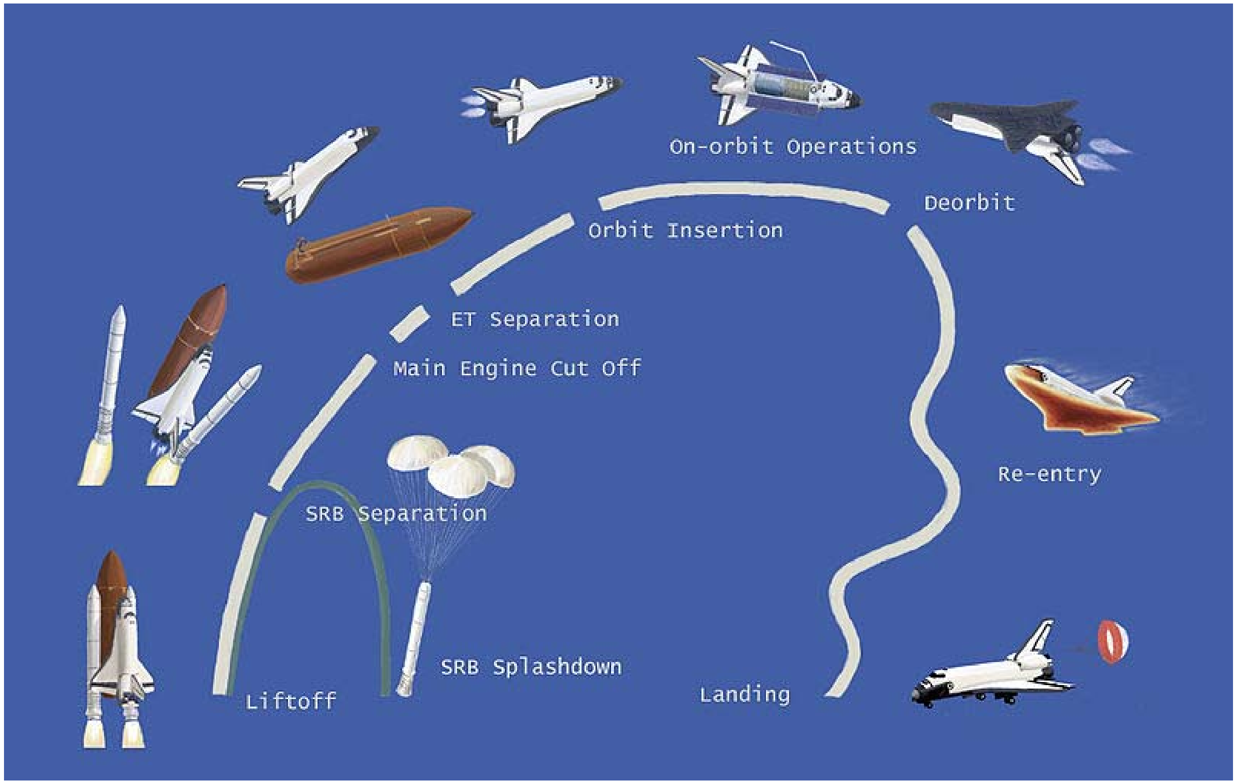
This is a modeling exercise:

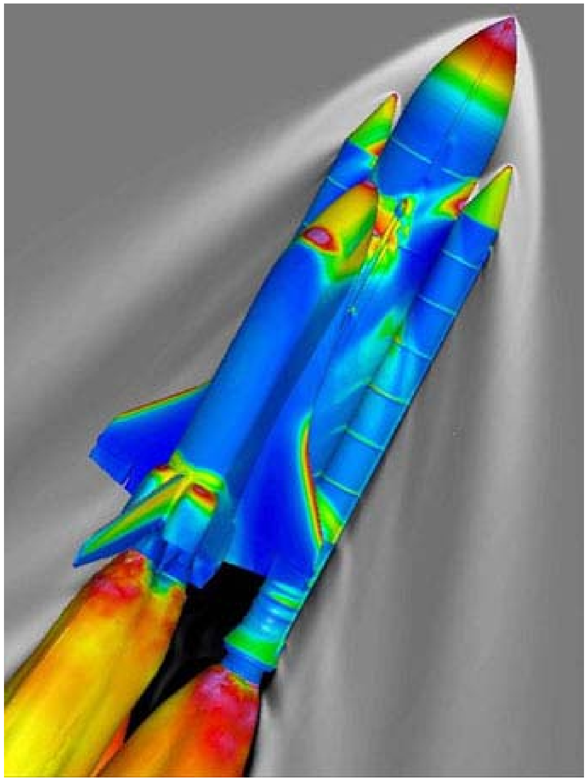
- Start simple (but we know it is a complex problem)
- Consider only what seems relevant
- Think of the physics involved

Key point:

Trying to understand things exactly,
we often miss the important fundamentals
and get hung up on side secondary issues.

NASA Space Shuttle Mission Profile





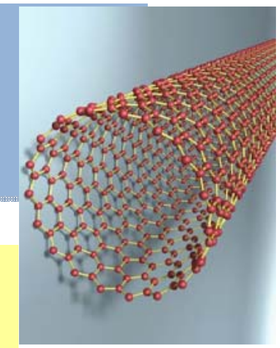
Tesla Car (when smaller is better)

- Code name: Dark Star
- Maker: Tesla Motors, Menlo Park, CA
- Engine: 3-phase electric motor (Li-ion battery technology)
- Performance: 0-60mph in 4 sec
- Max speed: 125mph
- Efficiency: 244 miles in a single battery charge
- Base price: $100k (but you get tax incentives!)

The engineering secret:

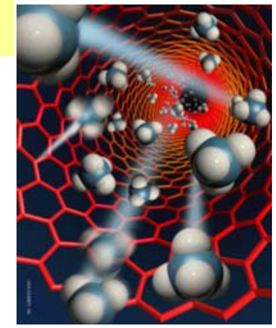
- 6800 Li-ion battery cells (just a bit smaller than regular AA battery)
- total battery pack ~ 5-10% of total car weight

Nanotechnology



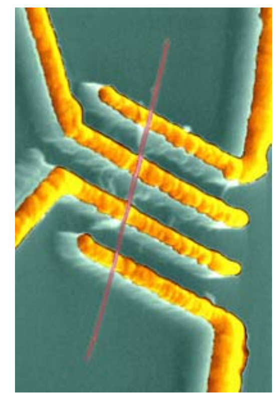
1 nanometer = 1/100,000,000 of a meter

(one billionth of a meter, or 10^{-9} meters, the size of a carbon atom)

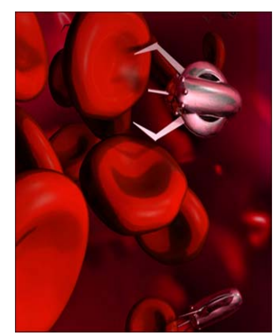


Nanotechnology is …

- An emerging, interdisciplinary science involving **mathematics**, physics, chemistry, biology, engineering, etc.;



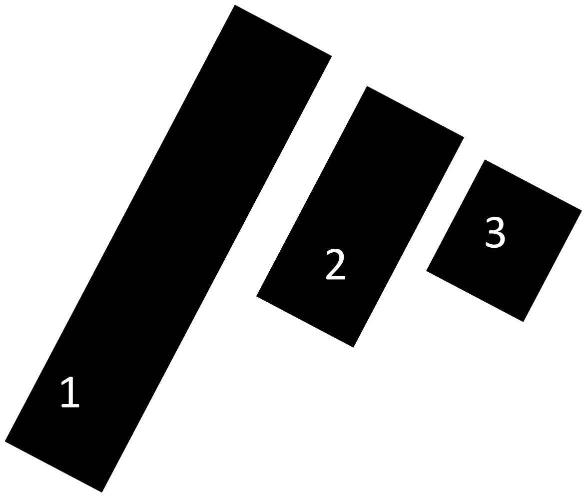
- A way to study, control and manipulate matter at the atomic scale (nano-scale);



- An *enabling technology* with impacts on electronics, computing, medicine, energy, environment, and much more!

On the cutting-edge
What does it take to work at the nanoscale.

- Materials
  - 1 strip of paper 11” x 1”
  - Scissors



- Questions
  - How many cuts can you make?
  - How many cuts would you need to make to reach the size of an atom (1nm) ?

Table for data organization

Cut #	Fraction of original strip	Power Rule	Length (cm)	Comparison
0	1		28	
1				
2				
3				
4				
5				
6				

Table for data organization

Cut #	Fraction of original strip	Power Rule	Length (cm)	Comparison
0	1	$1/2^0$	28	My arm
1	1/2	$1/2^1$	14	My hand
2	1/4	$1/2^2$	7	Index finger
3	1/8	$1/2^3$	3.5	
4	1/16	$1/2^4$	1.75	
8	1/256	$1/2^8$	0.11	

Challenge #3: On the Cutting-edge

How many cuts are needed to reach 1nm?

What I know: for 8 cuts I have

$0.109 \text{ cm} = 28 \text{ cm} \times 1/2^8$

What I need to know: after x cuts I have 1nm

$$10^{-7} \text{ cm} = 28 \text{ cm} \times 1/2^x$$

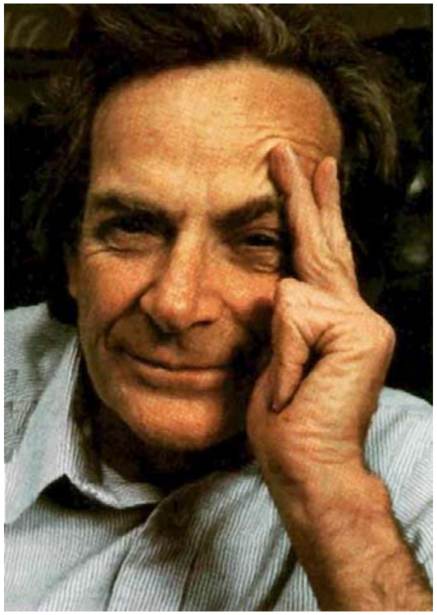
Need to solve for x: $1/2^x = 10^{-7}/28$

You can solve this in 2 ways:

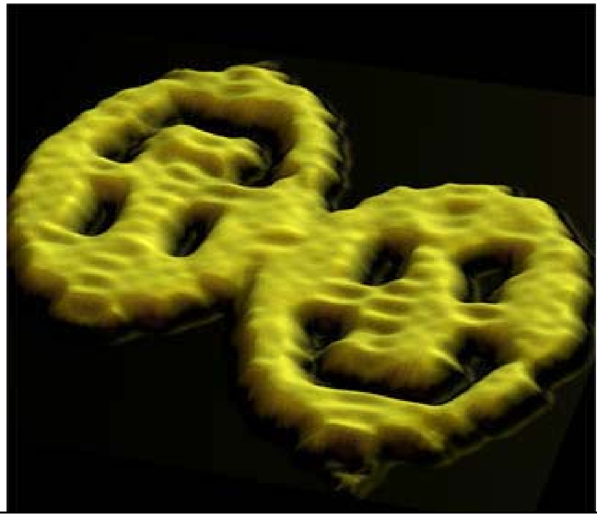
- exactly (using log and calculator) x = 28
- or by approximations ($28 = 32 = 2^5$ and $5 = 4 = 2^2$ ➔ $10^7 = 2^{21}$ or $5 = 8 = 2^3$ ➔ $10^7 = 2^{28}$) x =26; x = 33

How did we get here?
Origin of Nanoscience

- New Tools! As tools change, what we can see and do changes.



First theoretical introduction by
Richard Feynman
"*There's Plenty of Room at the Bottom*"
American Physical Society, 1959



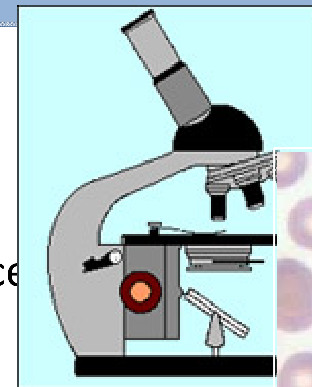
DNA Origami by Paul Rothemund

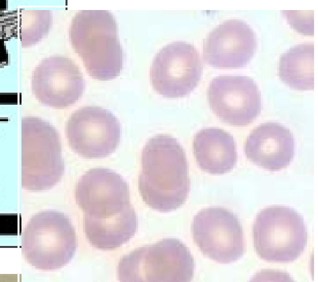
The key is to make use of the unique properties which arise because of the nanoscale.

How to See Small Things

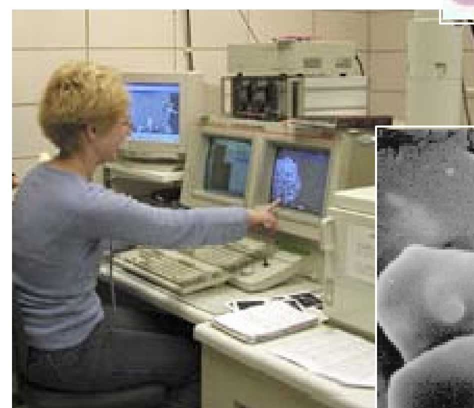
- The **naked eye** can see to about 20µm

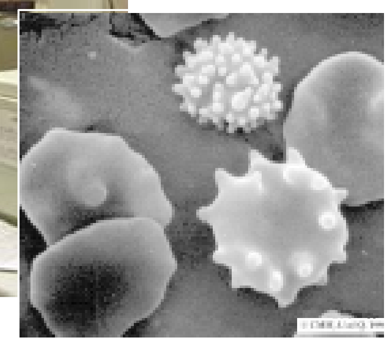
- **Light microscopes** let us see to about 1µm (bounce **light** off of surfaces to create images)



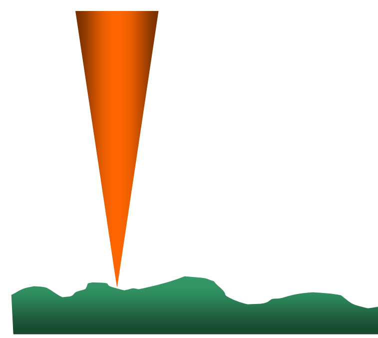


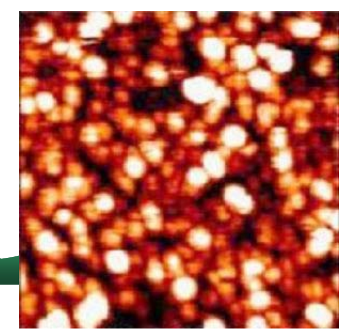
- Scanning electron microscopes (1930s) let us see objects as small as 10 nm (bounce electrons off of surfaces to create images)



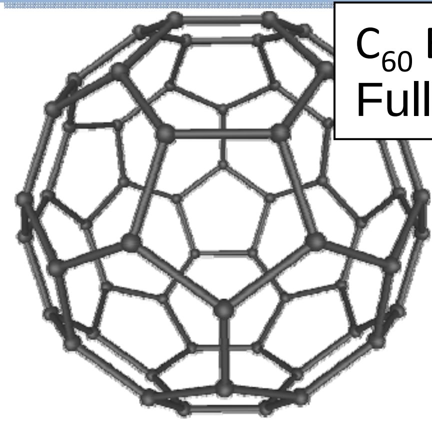


- Scanning tunneling microscope (1980) and atomic force microscope (1986) let us see and interact with the atomic structure (use the atom-to-atom interaction to map the surface)





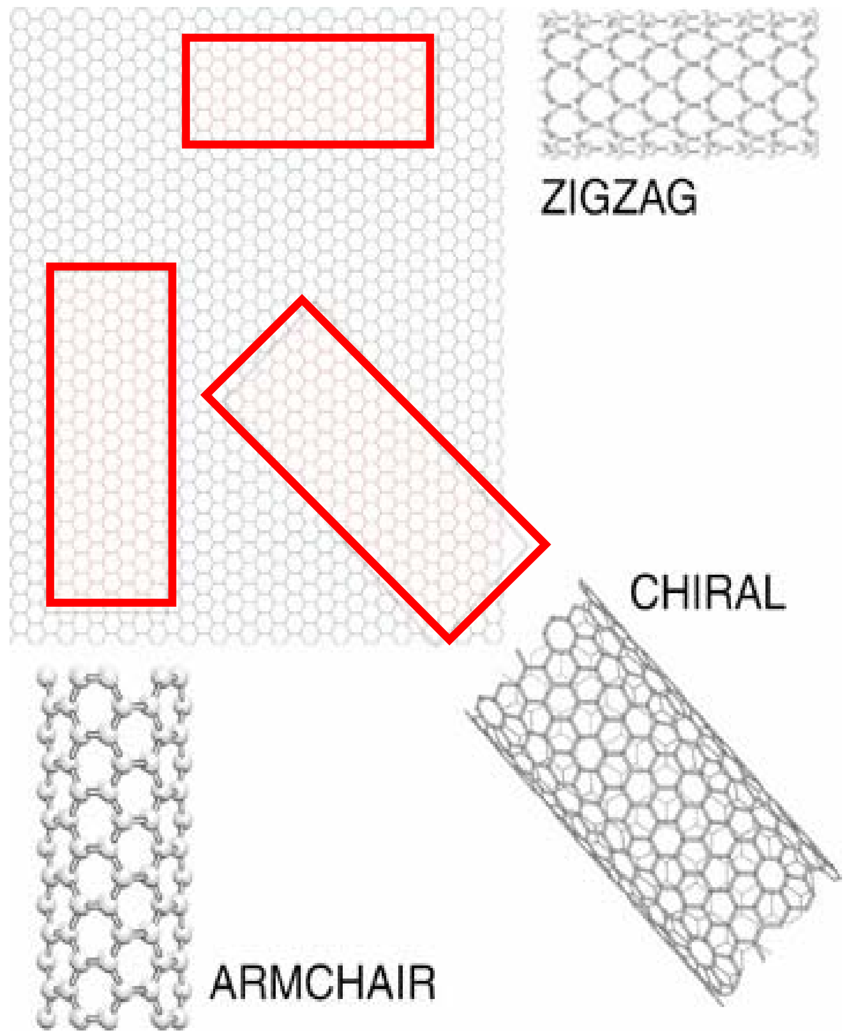
What did we see? New things!



C_{60} Bukyball or Fullerene

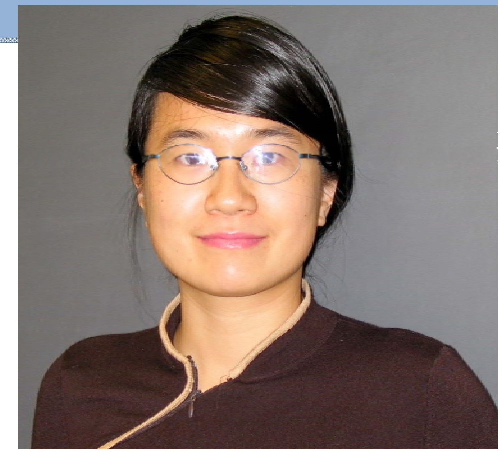
Fuller's American Pavillion, Monreal Expo '67

Graphene and Nanotubes

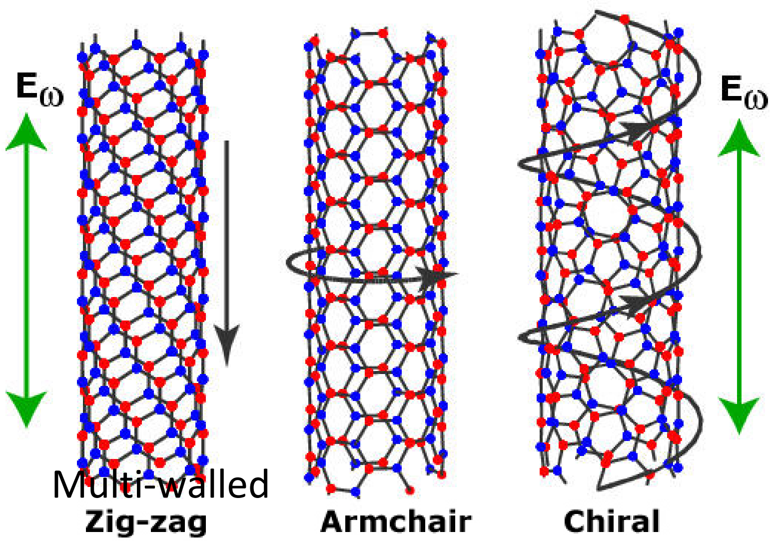


Electrical Properties: Conductivity of Nanotubes

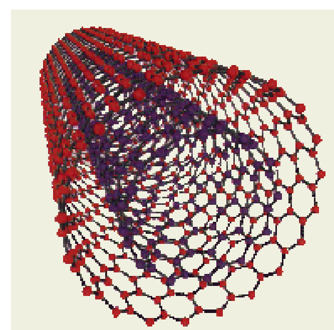
- Because the mass of nanoscale objects is so small, gravity becomes negligible
- Nanotubes are long, thin cylinders of carbon
  - They are 100 times stronger than steel, very flexible, and have unique electrical properties
- Their electrical properties change with diameter, "twist", and number of walls
  - They can be either conducting or semi-conducting



Jeanie Lau



Multi-walled

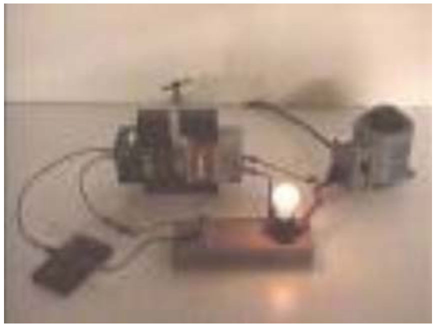


Source: http://www.weizmann.ac.il/chemphys/kral/nano2.jpg

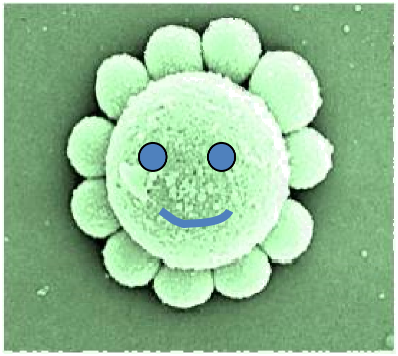
Size-Dependent Properties of Materials

- Types of properties that describe how materials act under certain conditions:
  - Optical (e.g. color, transparency)
  - Electrical (e.g. conductivity)
  - Physical (e.g. hardness, melting point)
  - Chemical (e.g. reactivity, reaction rates)

- At the nanoscale, properties change because different forces dominate at different scales.



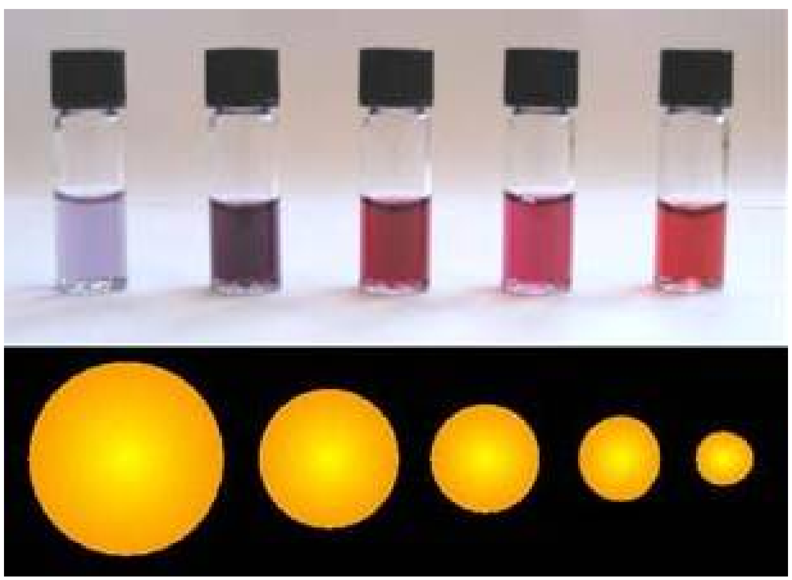
- If you are a nanoparticle you are:
  - *Sticky* (intermolecular forces)
  - *Shaky* (thermal energy)
  - *Bumpy* (quantum effects)
  - *Gravity doesn't matter (really weak)*

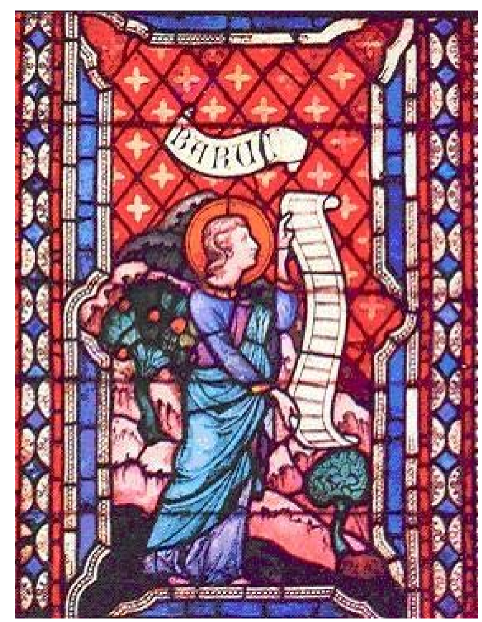


Optical Properties: Color of Gold

Nanosized gold appears red in color

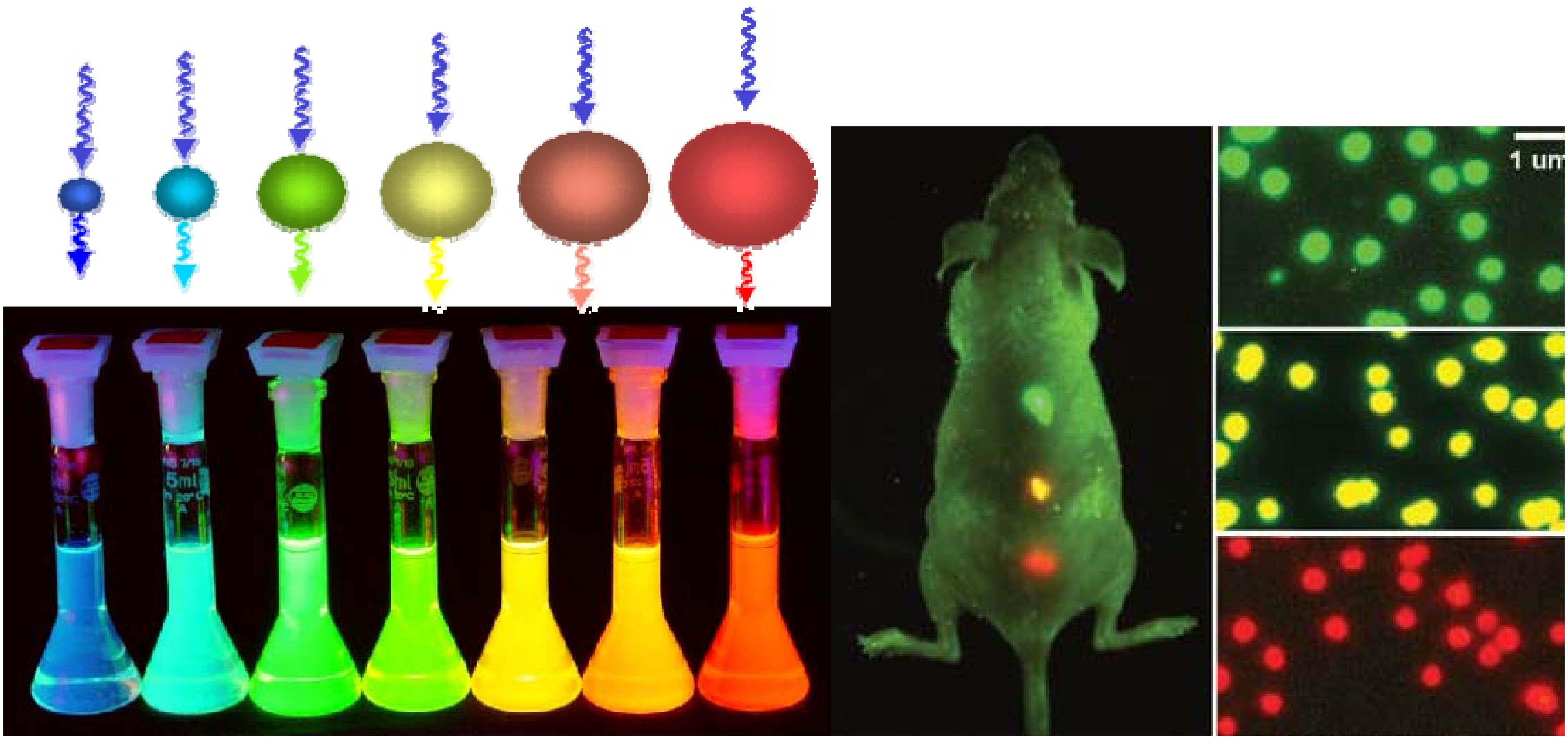
- Nano particles are so small that electrons are not free to move about as in bulk gold
- Because the movement is restricted, the particles react differently with light



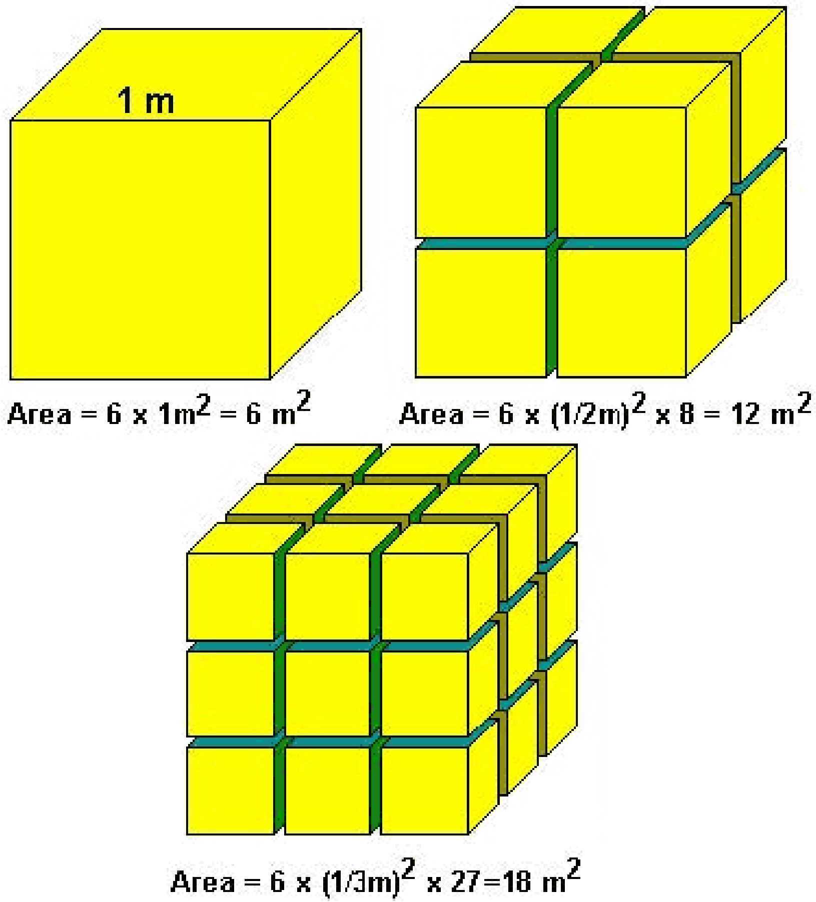


Optical Property of Quantum Dots

Quantum dots (metal oxide particles) glow in UV light and their color depends on their size



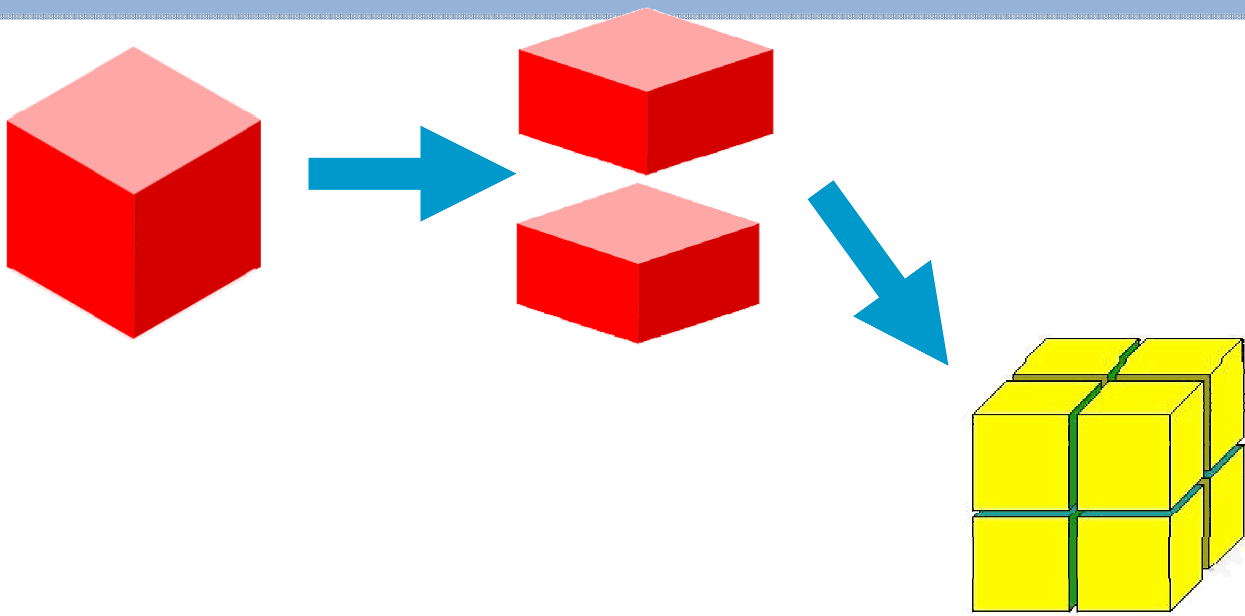
It's all at the surface…



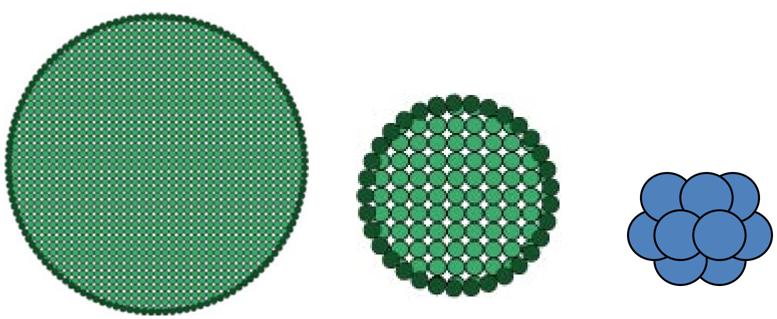
Nanomaterials have a large surface area for a given volume (increase of surface to volume ratio)

	At the macroscale	At the nanoscale
The majority of the atoms are…	…almost all on the inside of the object	…split between the inside and the surface of the object

Technology: It’s all at the surface…

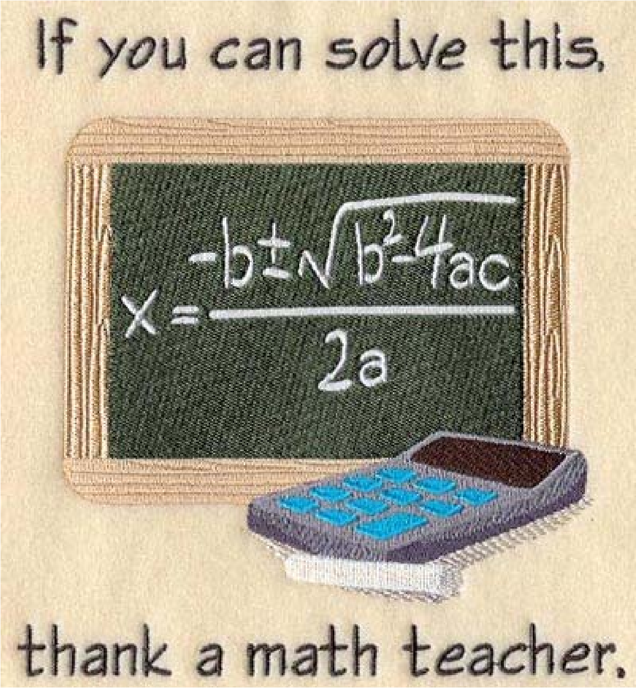


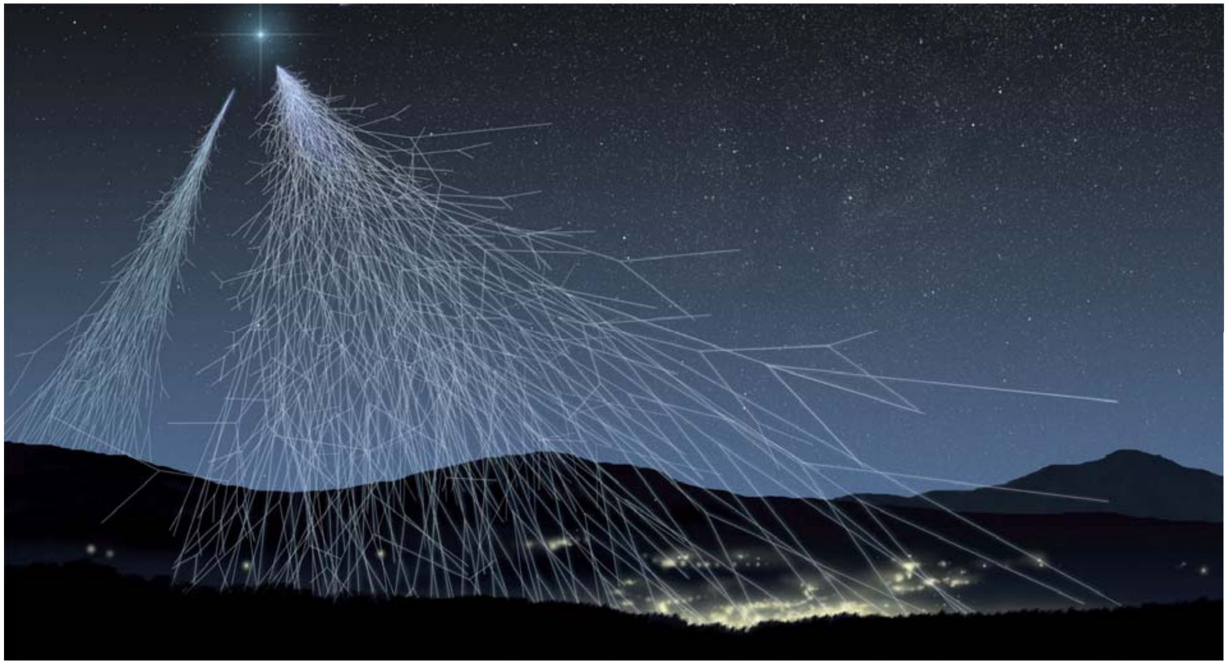
Sharon Walker



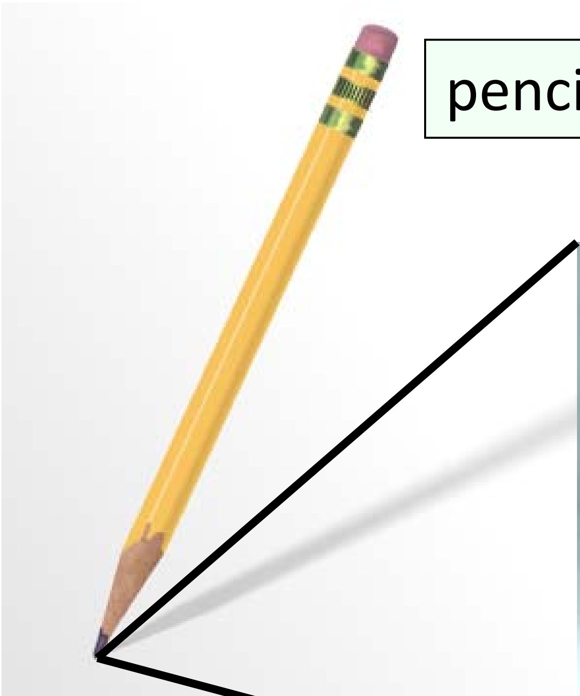
Efficient water filters

Discovery of Antimatter

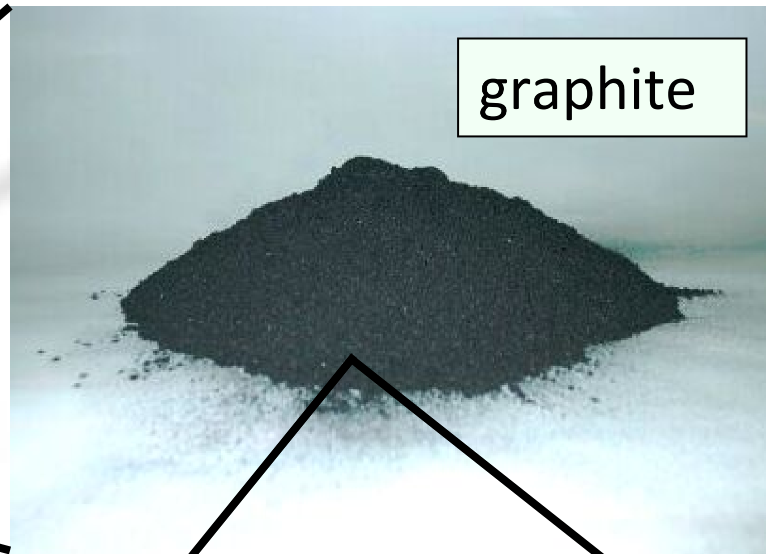


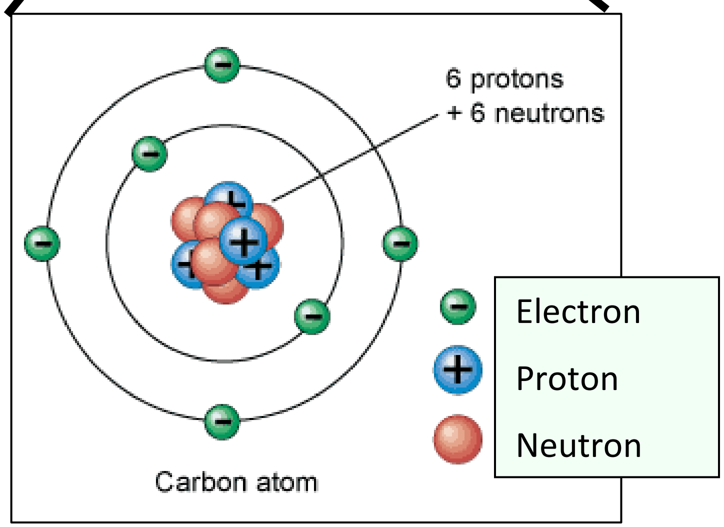


What is matter made of?



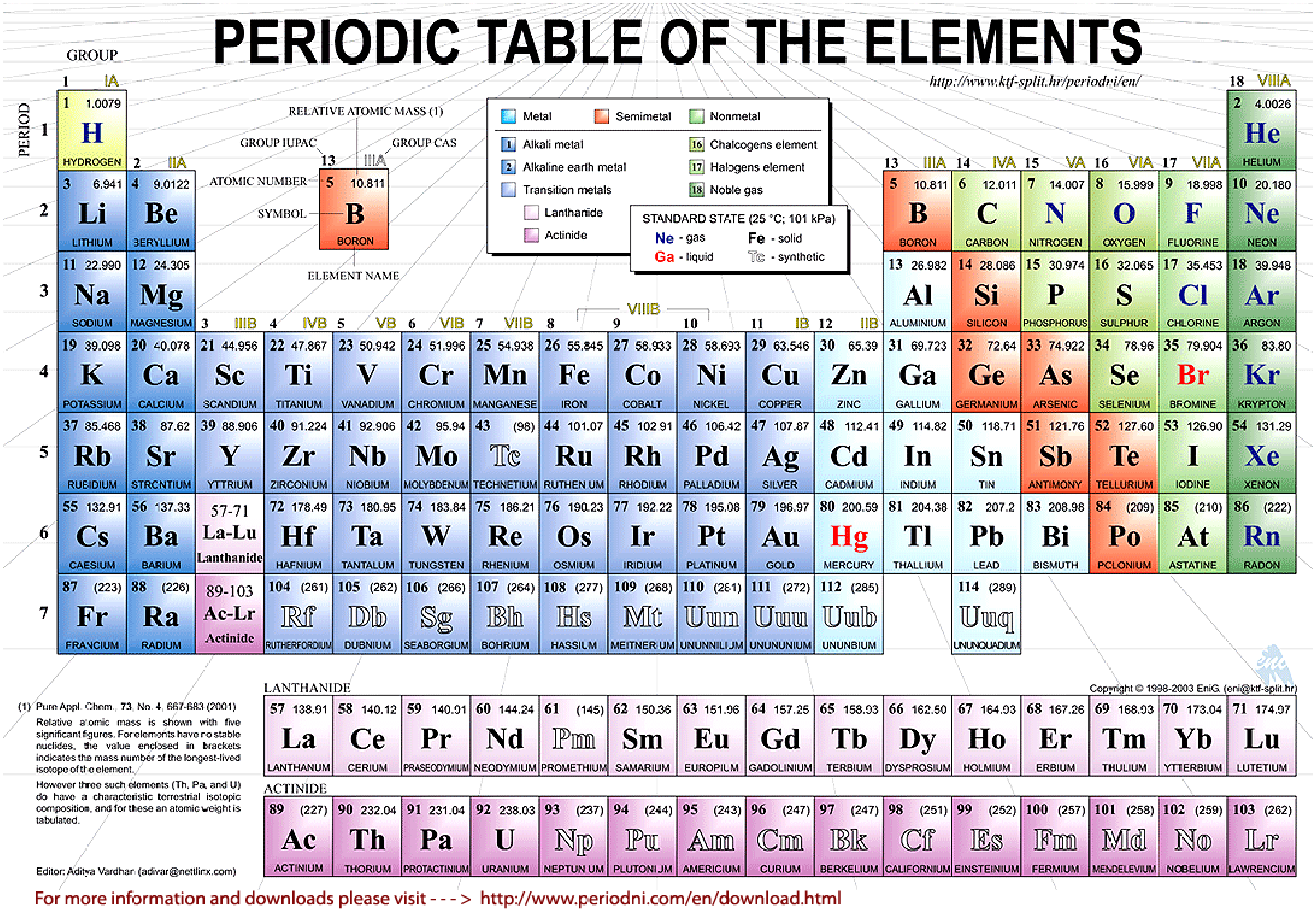
pencil



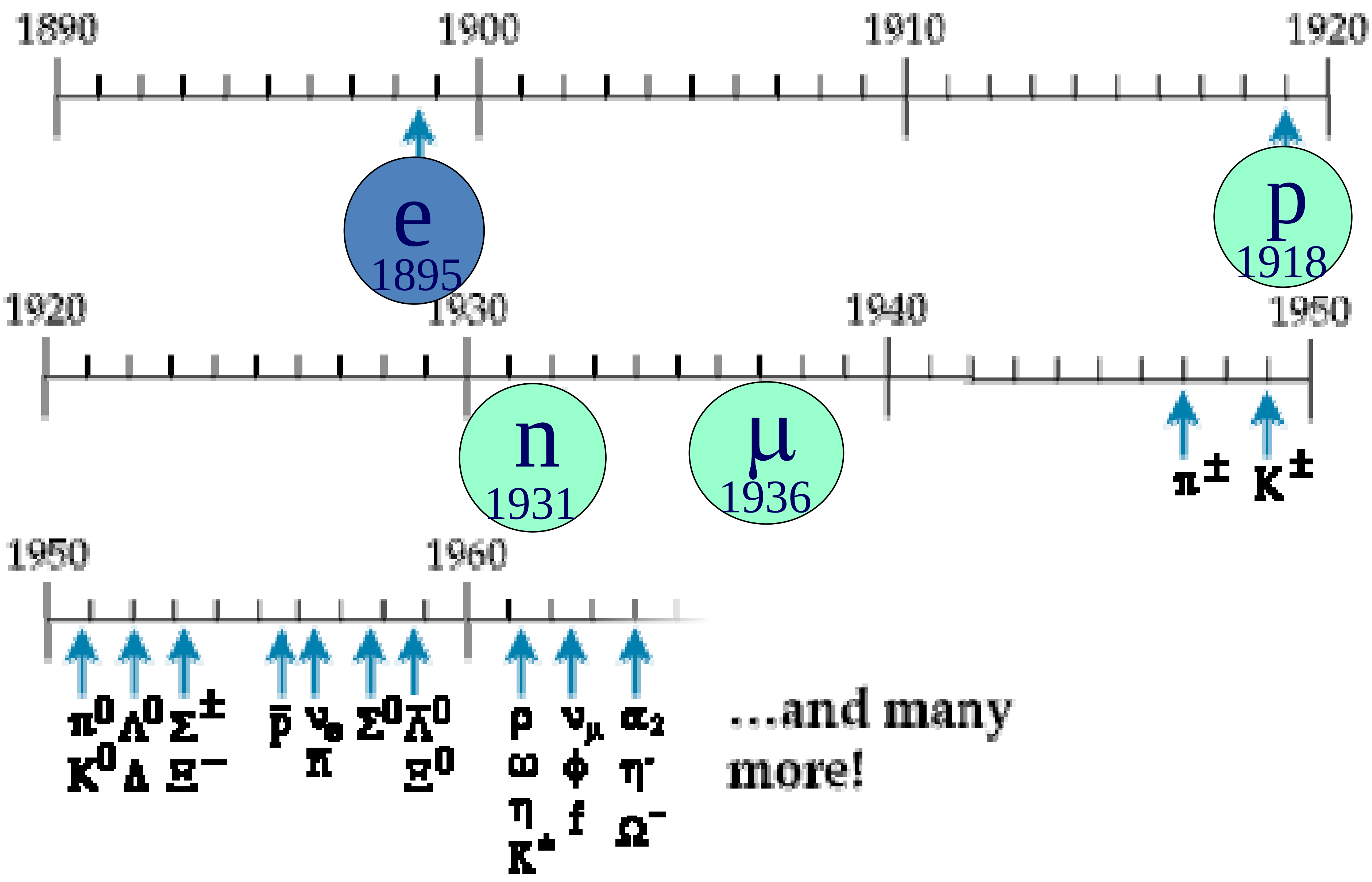


carbon atom

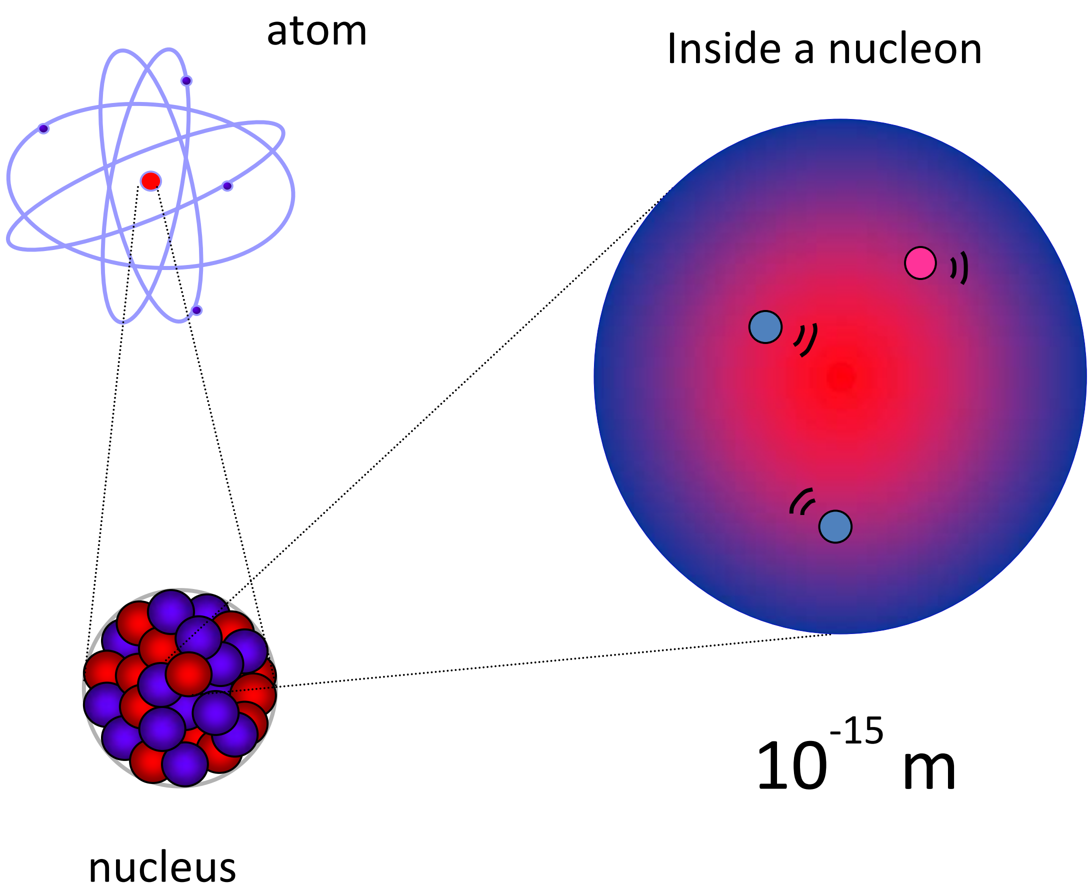
Protons, neutrons and electrons are the building blocks for all the elements we know



New particles are discovered



Is there more? The structure of the nucleus

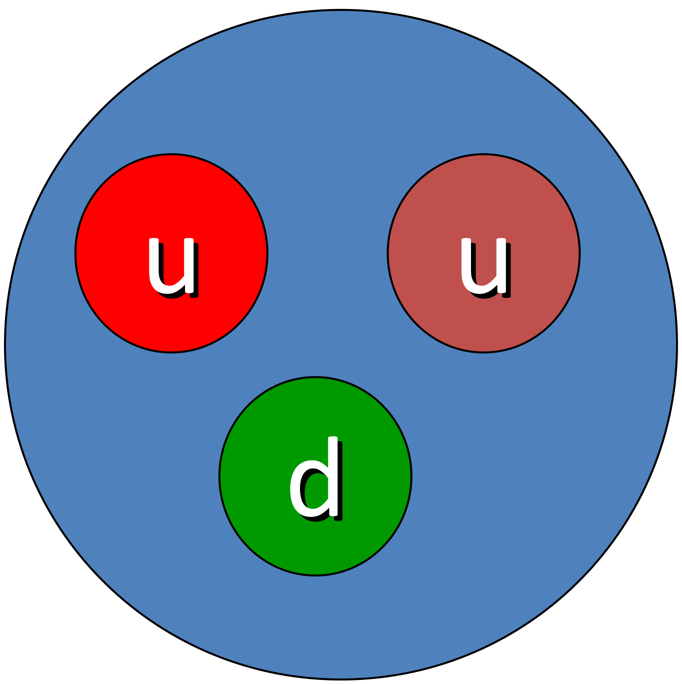


Neutrons and protons contain *quarks*

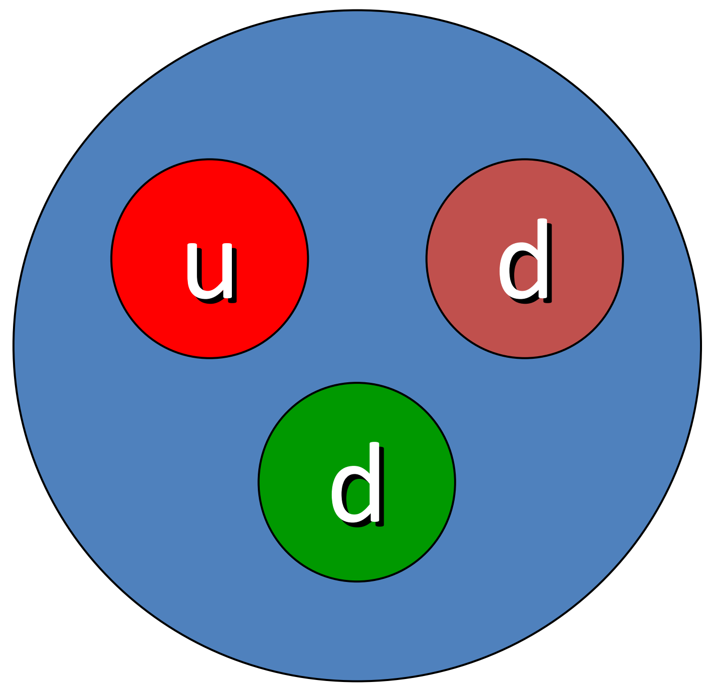
Protons and neutrons in the quark model

The quark hypothesis was tested at the Stanford Linear Accelerator Center (SLAC) in California, late 1960s

proton (charge +1)

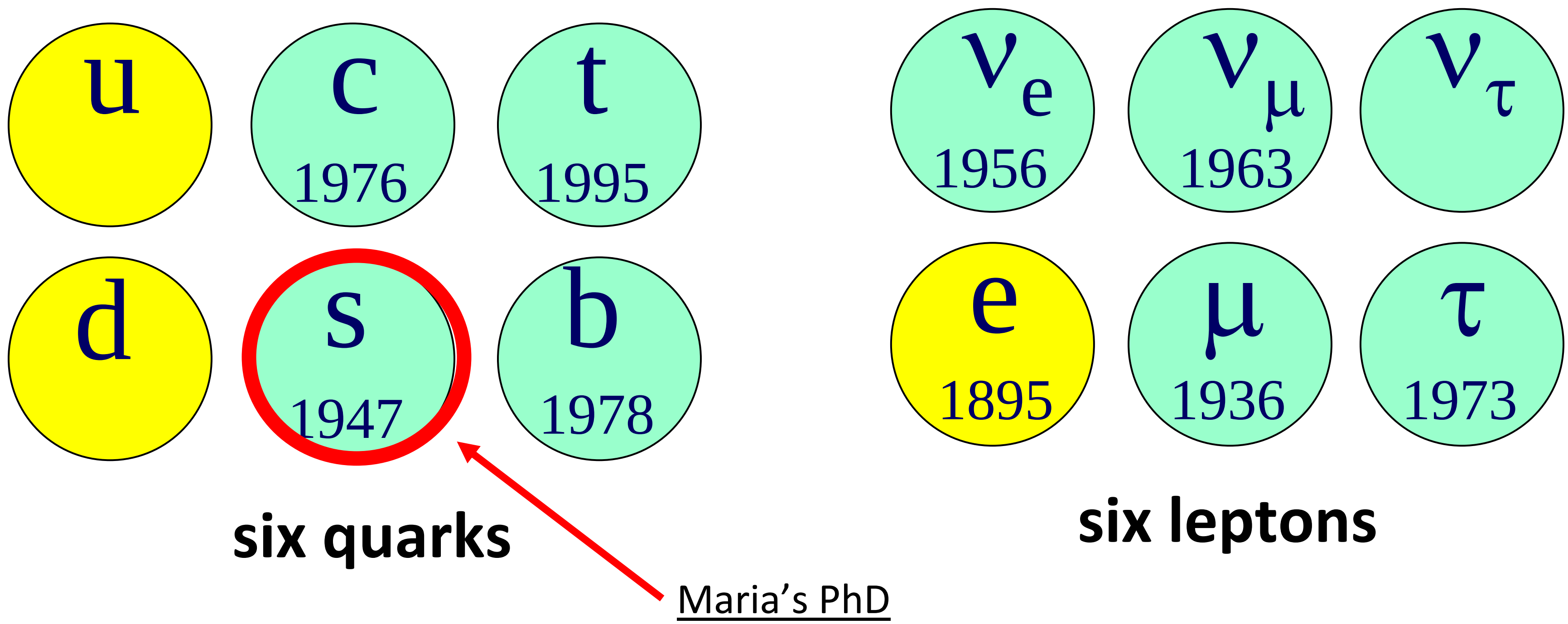


neutron (charge 0)



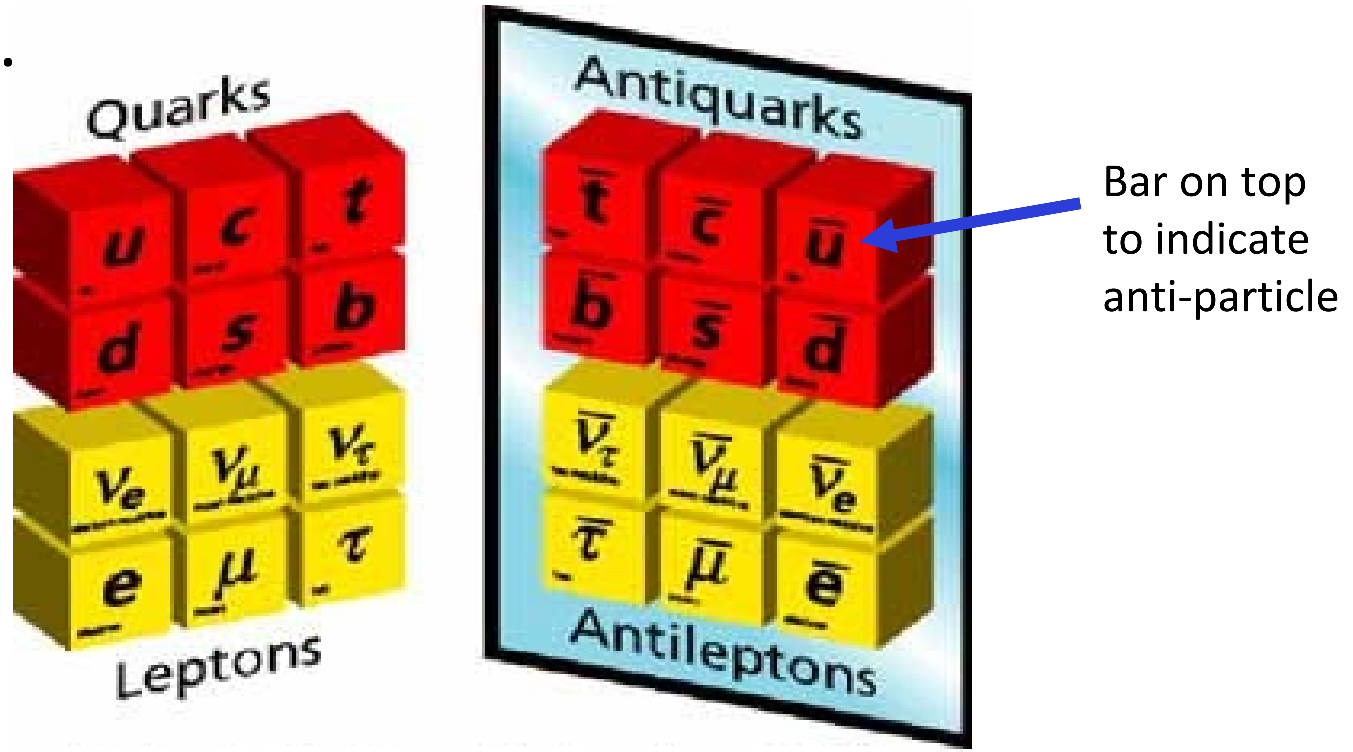
The zoo grows larger

We believe these to be the fundamental building blocks of matter



Is that all?

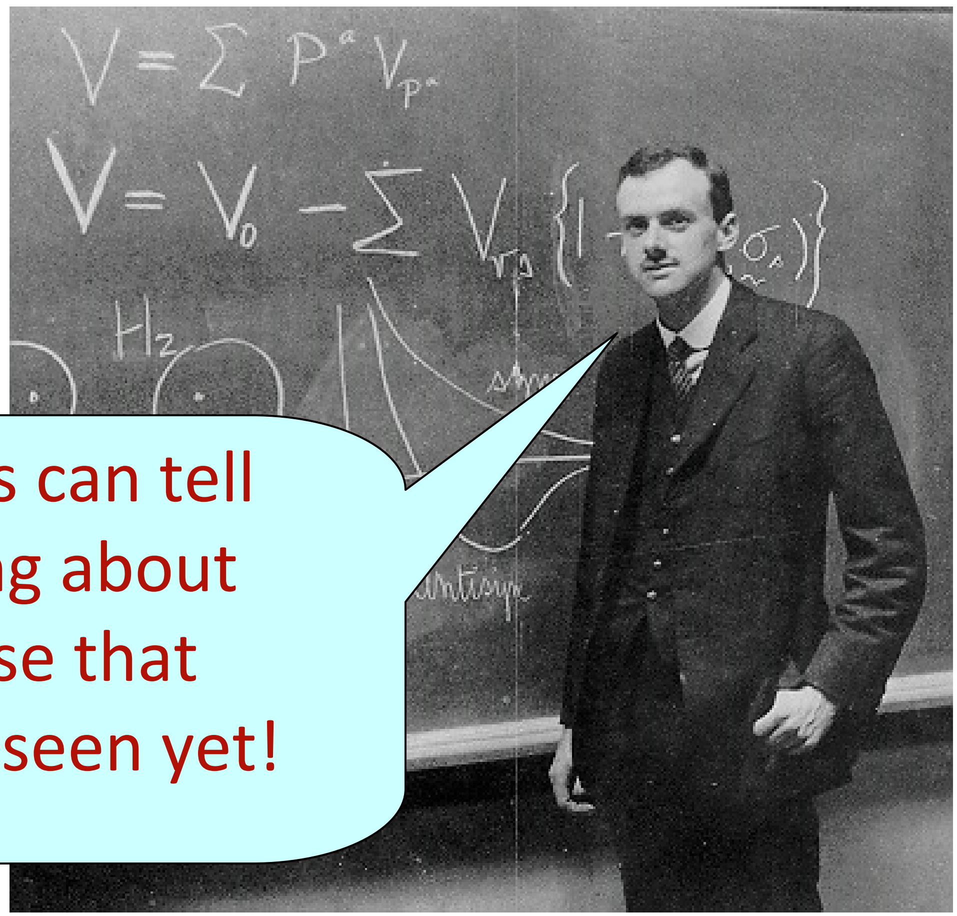
Actually…no.



- For every quark, there is a corresponding <u>anti-quark</u>
- For every lepton, there is a corresponding anti-lepton
- They look just like matter but have the opposite charge

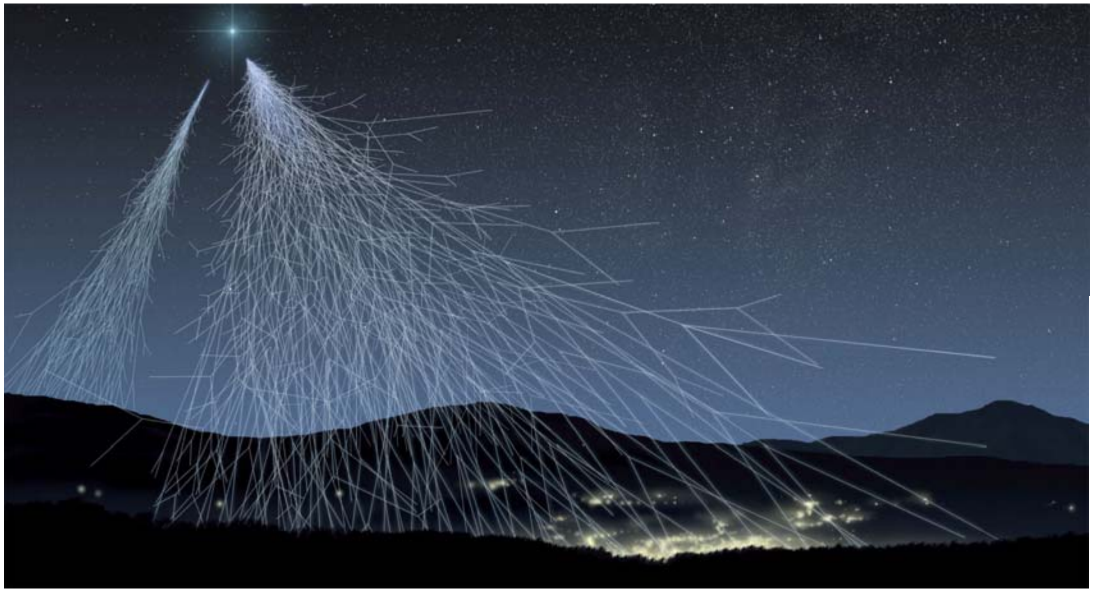
Prediction of antimatter

- In 1928 the young Paul Dirac mathematically predicted the existence of antimatter (the positron) when solving a quadratic eequation.
- In 1932 Anderson discovered the positron predicted by Dirac.

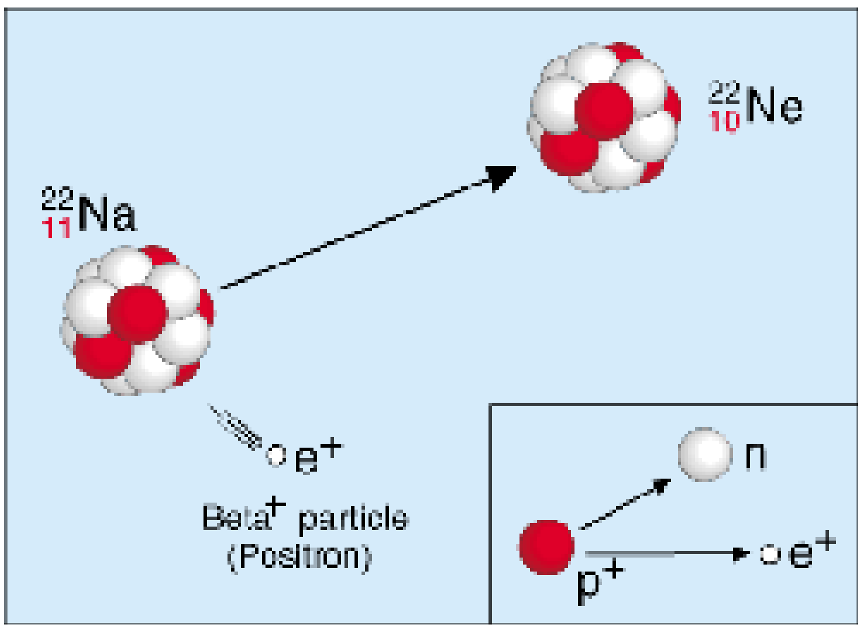


Mathematics can tell
us something about
our universe that
we have not seen yet!

Nature can make anti-particles



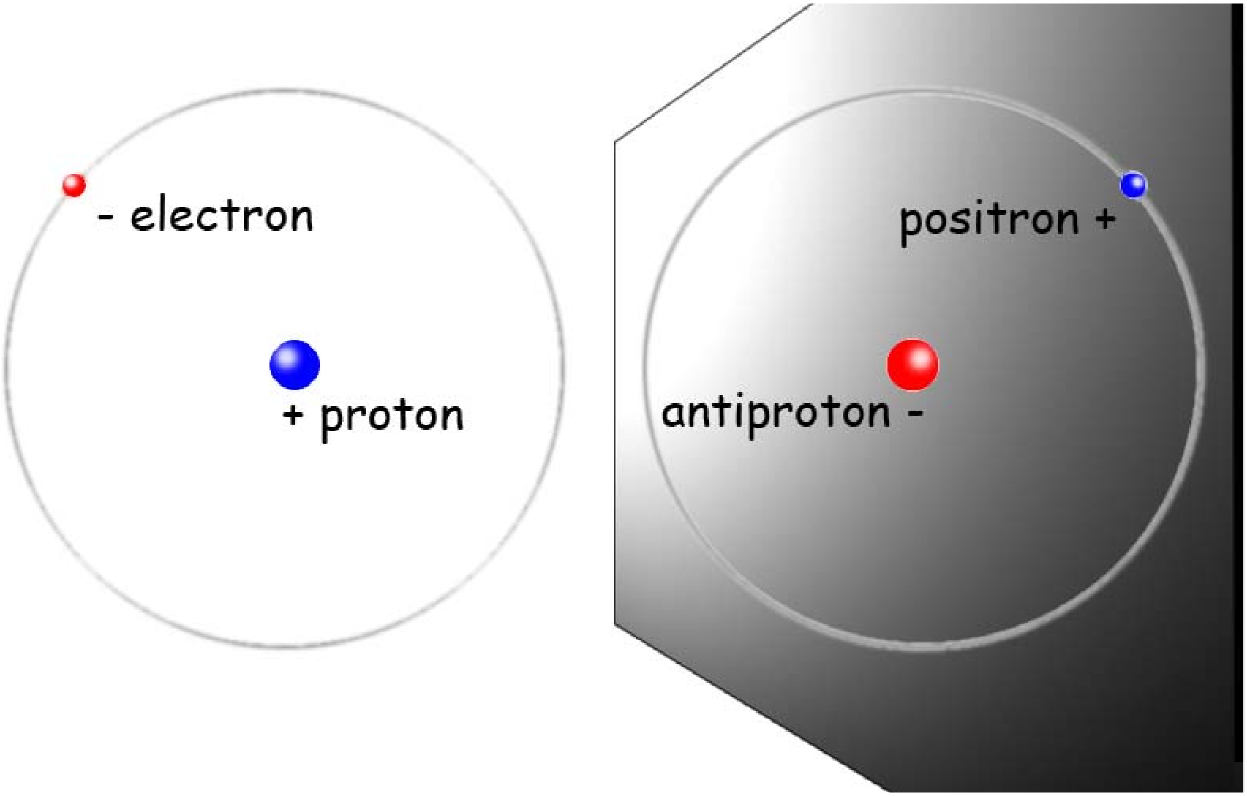
Cosmic rays are made up of many antiparticles



Radioactive beta decay:
Sodium = Neon + e⁺

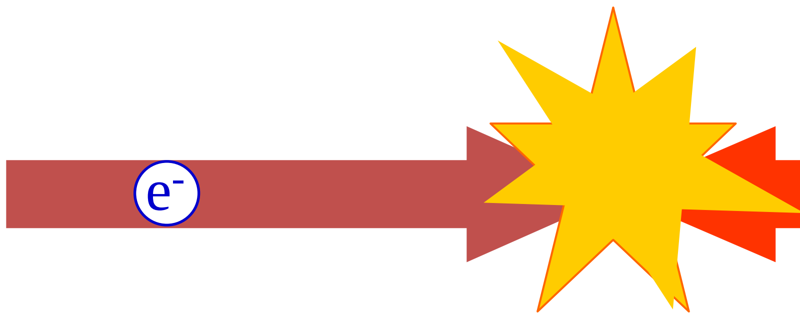
We can make anti-Hydrogen

- If you put a positron and an antiproton together you get a molecule of anti-hydrogen
- This is what they can make at CERN in Switzerland and at Fermilab in the USA.



An explosive relationship

- When an antimatter particle touches a matter particle they are both wiped out in a flash of energy
- At CERN they can only keep positrons and antiprotons briefly in an elaborate magnetic trap

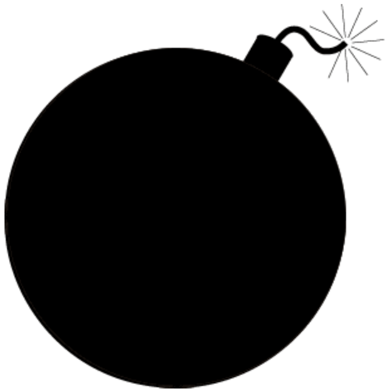


Very small energy

What can we do with anti-matter?

Can we make anti-matter bombs?

- There's no chance of making an antimatter bomb.
- It takes way more energy to make mere particles of the antimatter than you get out when it's annihilated with matter!

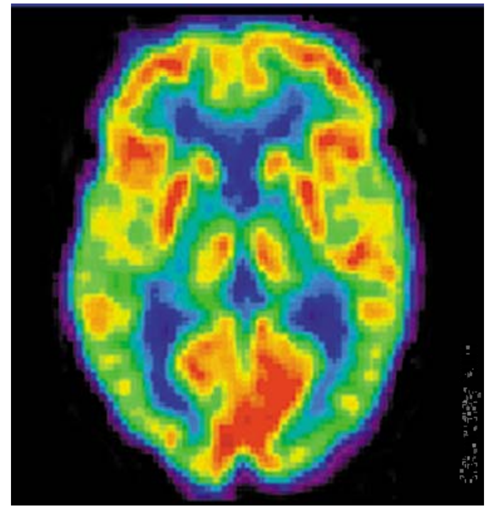


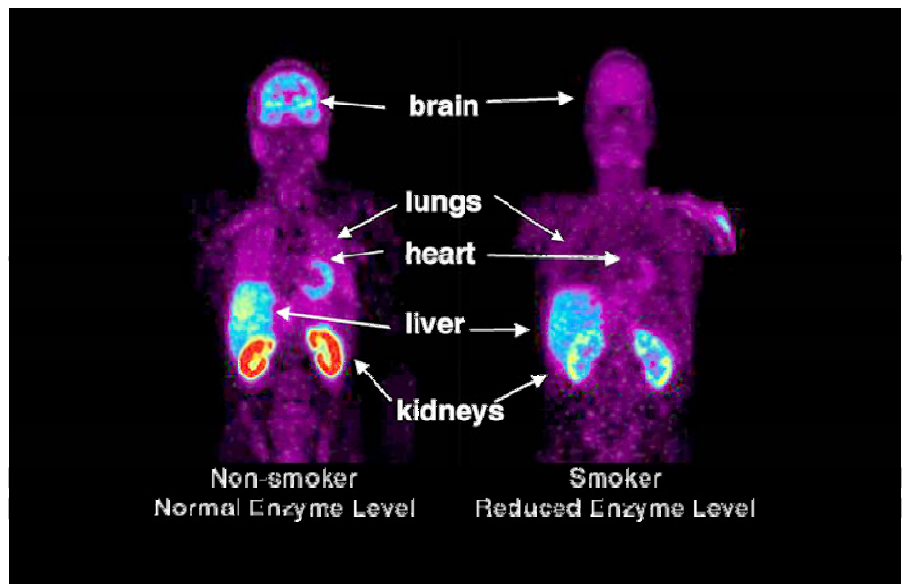
Can we use it for energy?

- Not really. In 10 years, CERN has made just about a billionth of a gram of antimatter
- Enough to power a light bulb for a few minutes
- More energy goes in than it is produced
- Maybe a new propulsion technique for spaceship...?

Antimatter is used in medicine

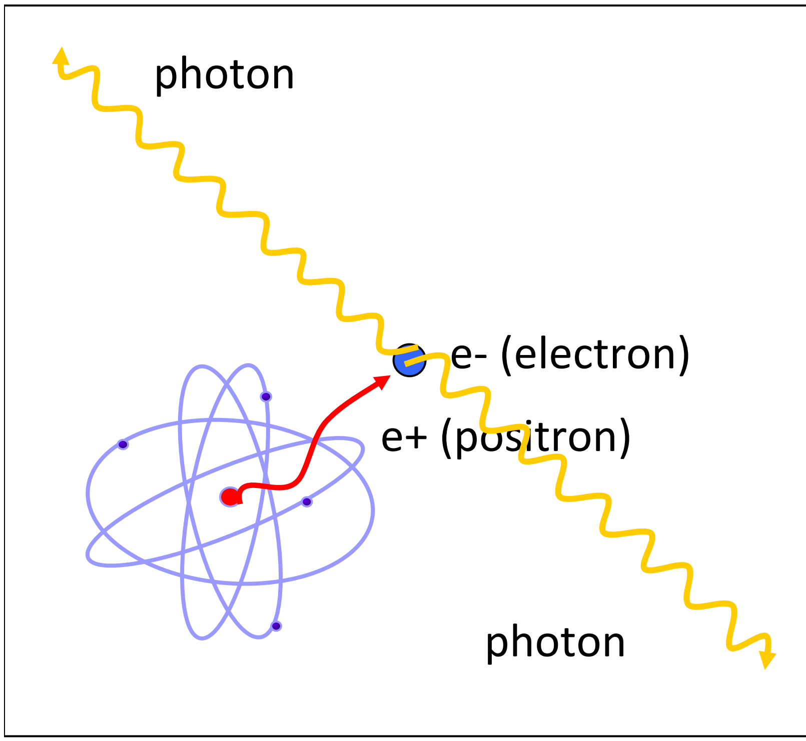
PET Scans
(Positron Emission Tomography)

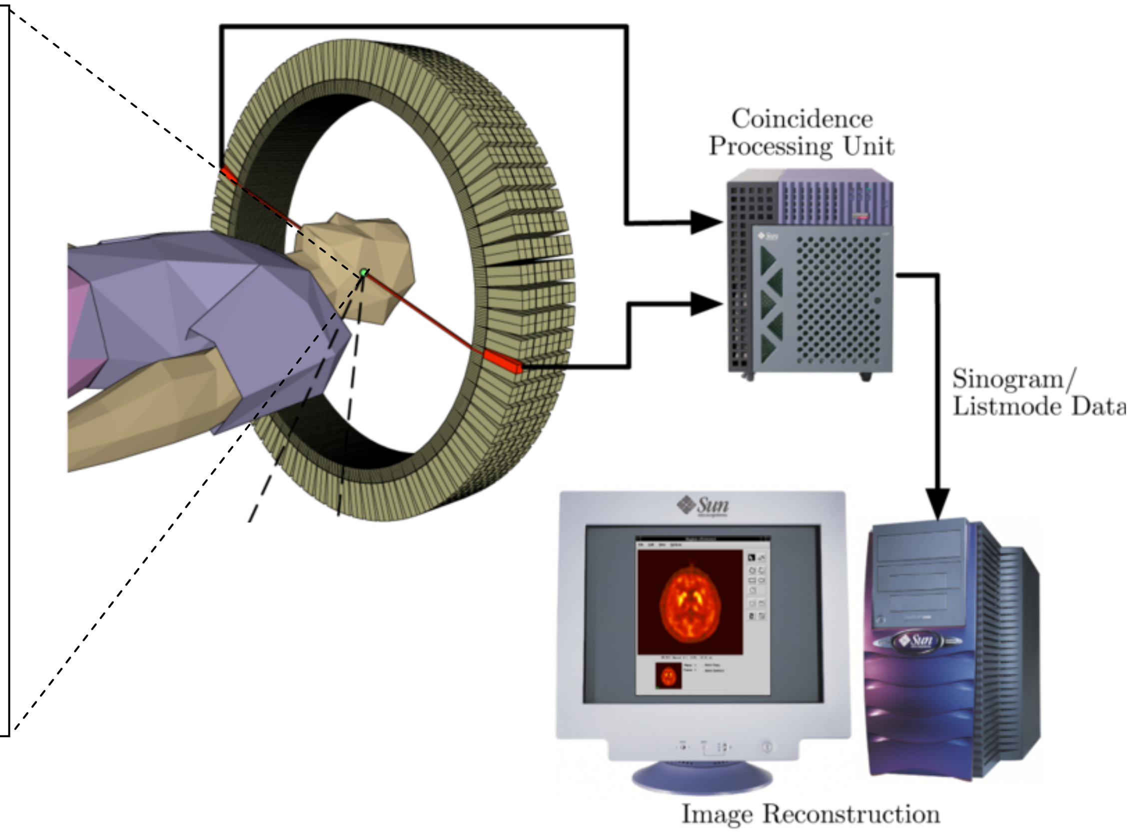




How does it work?

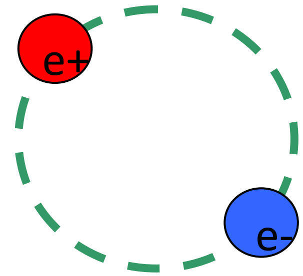
Positron emission tomography (PET) is a nuclear medicine imaging technique which produces a 3D image of functional processes in the body.





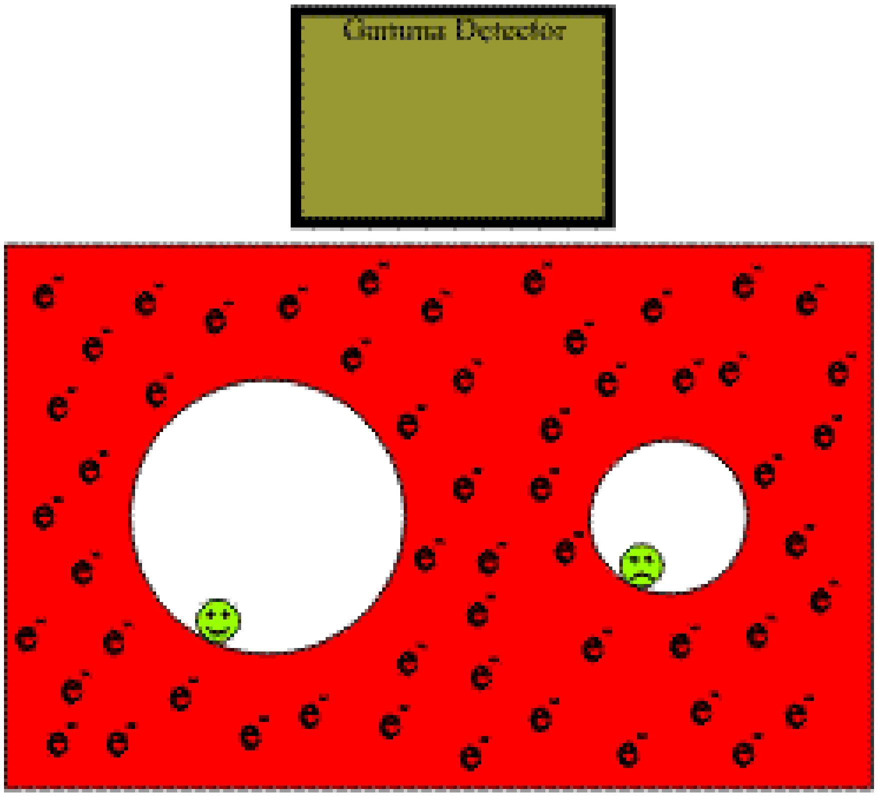
What else can we do with positrons?

We can bind an electron to a positron to make *positronium* *(UCR has a leading research team)*



Applications of positronium include:

- Intense Gamma-Ray Laser
- Positronium Annihilation Lifetime Spectroscopy to probe the microscopic structures of materials
- *[Your application here]*



This is where I spend some of my time now…

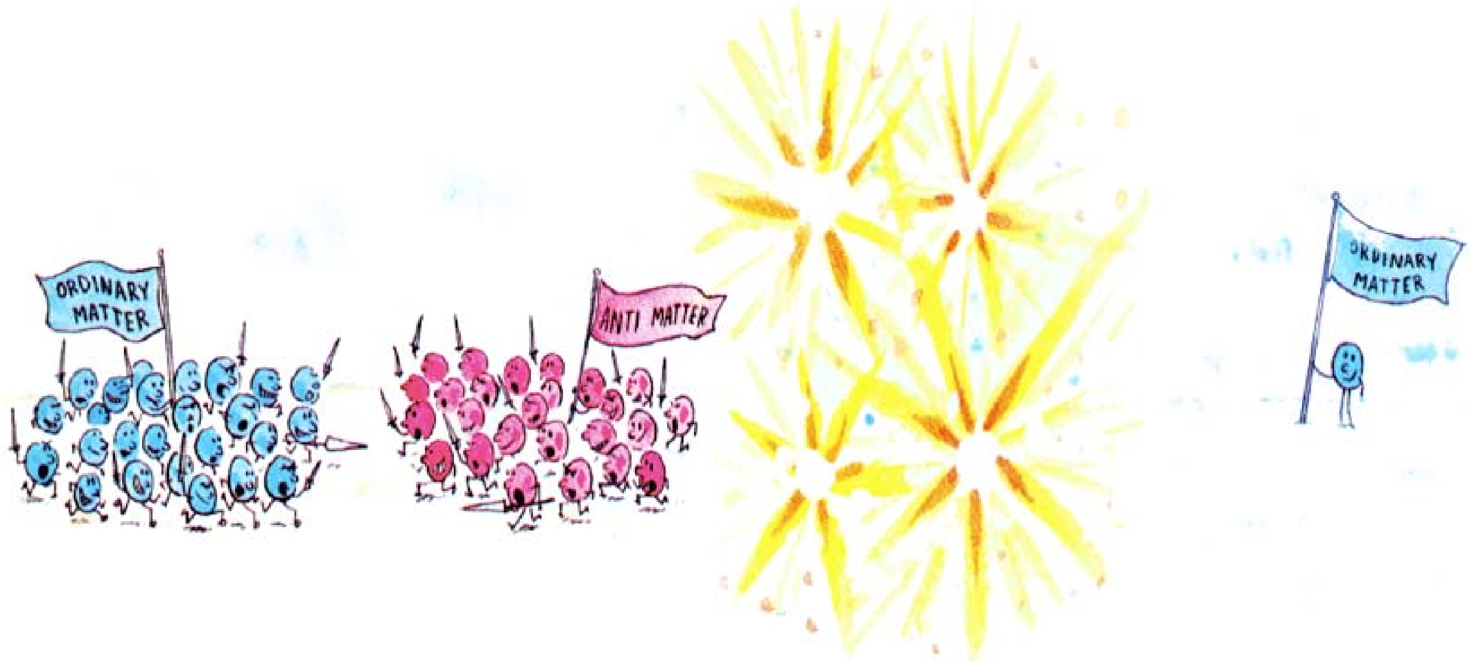
Let’s do some thinking

- Anti-matter exists: nature can make it and we can make it too.
- For each matter particle we create, an anti-matter particle is created too.
- But, there is very little natural anti-matter in the universe…

- Where is all the anti-matter?
- Does anti-matter behaves like matter?
- Can we use anti-matter?

Matter, Antimatter, and the Universe

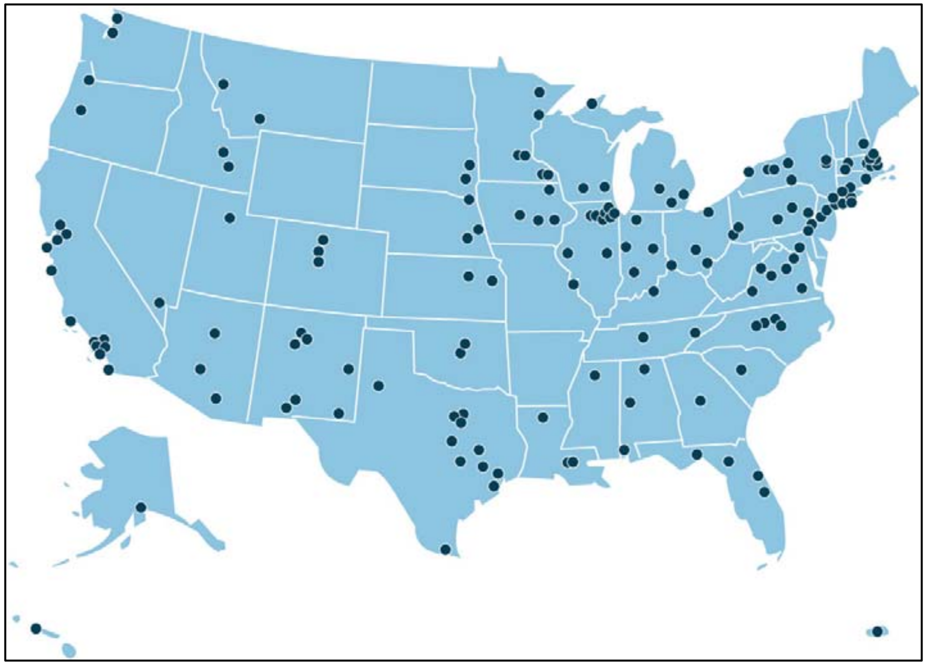
- At the beginning of the Universe, 15 billion years ago, the Big Bang produced equal amounts of matter and anti-matter
- Everything should have annihilated. Instead…
- Where did all the anti-matter go?



In the early universe, for every billion ordinary particles annihilating with antimatter, one was left standing…

Searching for Answers

- Particle physics laboratories around the world



- At universities and laboratories across the United States

Summary: what is important to know?

Mathematical Practices

1. Make sense of problems and persevere in solving them;
2. Reason abstractly and quantitatively;
3. Model with mathematics;
4. Look for and express regularity in repeated reasoning.

The truth may be puzzling. It may be counterintuitive. It may contradict deeply held prejudices. It may not be consonant with what we desperately want to be true.

But our preferences do not determine what's true. We have a method, and that method helps us to reach not absolute truth, only asymptotic approaches to the truth — never there, just closer and closer, always finding vast new oceans of undiscovered possibilities.

Carl Sagan